Sheet (2)

The cube root of a rational number

Example (1):

Find the area of a square whose side length 5 cm?

Area = $S \times S = S^2 = 5^2 = 25$ cm^2.

The square numbers	The square root
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$\sqrt{x^4} = x^2$	$\sqrt{x^6} = x^3$

Example (2):

Find the volume of a cube whose edge length 5 cm?

$V = S \times S \times S = S^3 = 5^3 = 125$ cm^3.

The cub numbers	The cube root
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^2 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$
$\sqrt[3]{x^3} = x$	$\sqrt[3]{x^6} = x^2$

[1] Complete the following table:

Number a	**8**	**125**	**-27**	.....	$3\frac{3}{8}$	$-\frac{8}{125}$	.....	.....
$\sqrt[3]{a}$	.....	.....	.....	**-10**	.....	.....	**6**	**-4**

[2] Find each of the following:

(1) $\sqrt[3]{216}$ = ...

(2) $\sqrt[3]{-343}$ = ...

(3) $\sqrt[3]{\frac{64}{125}}$ = ...

(4) $\sqrt[3]{\frac{-8}{27}}$ = ...

(5) $\sqrt[3]{0.001}$ = ...

(6) $\sqrt[3]{-2\frac{10}{27}}$ = ...

(7) $\sqrt[3]{8x^3}$ = ...

(8) $\sqrt[3]{-27a^6}$ = ...

[3] Complete:

(1) $\sqrt[3]{x^3}$ = ...

(2) $\sqrt[3]{.....}$ = 4

(3) $\sqrt{16} = \sqrt[3]{.....}$

(4) $\left|\sqrt[3]{-125}\right| = \sqrt{.....}$

(5) $\sqrt[3]{8} + \sqrt[3]{-8}$ = ...

(6) $\sqrt[3]{27} - \sqrt[3]{64}$ = ...

(7) $\sqrt[3]{27} - \sqrt[3]{-27}$ = ...

(8) $\sqrt{9} + \sqrt[3]{-8}$ = ...

(9) $\sqrt{64} - \sqrt[3]{64}$ = ...

(10) $-\sqrt[3]{-1} - \sqrt{1}$ = ...

(11) $\frac{-\sqrt[3]{64}}{\sqrt{64}}$ = ...

(12) $\sqrt[3]{64} = \sqrt{.....}$

(13) $\sqrt[3]{64+.....}$ = 5

[4] Find the value of x in each of the following:

(1) $\sqrt[3]{x} = 5$ $x =$ .....

(2) $\sqrt[3]{x} = \frac{-1}{4}$ $x =$ .....

(3) $\sqrt[3]{x} = -\sqrt{4}$ $x =$ .....

(4) $\sqrt[3]{x} - 3 = -1$ $x =$ .....

(5) $x^3 = -8$ $x =$ .....

(6) $x^3 = 64$ $x =$ .....

(7) $x^3 + 5 = 32$ $x =$ .....

(8) $2x^3 = 54$ $x =$ .....

(9) $\frac{1}{5}x^3 = -200$ $x =$ .....

[5] Choose the correct answer from those given:

(1) $\sqrt[3]{(-8)^2}$ = ... (2 , -2 , 4 , -4)

(2) $\sqrt[3]{-64} + \sqrt{16}$ = ... (0 , 8 , -8 , ±8)

(3) $\sqrt{25} - \sqrt[3]{-125}$ = ... (10 , 0 , 5 , ±5)

(4) $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3}$ = ... (-4 , 8 , 4 , 0)

(5) $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25}$ = ... ($\frac{3}{2}$, $\frac{1}{2}$, 2 , -2)

(6) $\sqrt[3]{x} = \frac{1}{4}$, then x = ... ($\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{12}$, $\frac{1}{64}$)

(7) If the volume of a cube is 64 cm^3, then the length of its edge = ... cm (8 , 4 , 32 , 16)

(8) If the capacity of a cubic vessel is 8 litres, then the length of its inner edge is ..... cm (2 , 4 , 20 , 40)

(9) If the volume of a sphere is 36 π cm^3, then the length of its diameter = ... cm (3 , 6 , 9 , 27)

(10) If $-\sqrt{25} = \sqrt[3]{y}$, then y = ... (5 , -5 , 125 , -125)

(11) If x^3 = 64, then $\sqrt{x}$ = ... (4 , -4 , 2 , -2)

(12) $\sqrt[3]{x^6} = \sqrt{.....}$ (x^3 , x^2 , x , x^4)

(13) If $\frac{x}{3} = \frac{9}{x^2}$, then x = ... (1 , 3 , 9 , 27)

[6] Find the S.S. of each of the following equations in Q:

(1) $x^3 + 27 = 0$

(2) $8x^3 + 7 = 8$

(3) $(x+3)^3 = 343$

(4) $(5x-2)^3 + 10 = 18$

(5) $2x^3 - 5 = x^3 + 3$

Sheet (3)

The Set of irrational numbers Q`

The set of irrational numbers denoted by Q` appear in:

(1) The square root of a non perfect square of a rational number such as: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ……

(2) The cube root of a non perfect cube of a rational number such as: $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$, ……

(3) $\pi \notin Q$

[1] In each of the following, show which of them is a rational number and which of them is an irrational number:

(1) -5

(2) $2\frac{2}{3}$

(3) 2.06

(4) 2.3×10^5

(5) $-\sqrt{36}$

(6) $\sqrt[3]{36}$

(7) Zero

(8) $\sqrt[3]{\frac{-64}{81}}$

(9) $\sqrt{\frac{1}{3}}$

(10) $\frac{\pi}{2}$

(11) $(-5)^{zero}$

(12) $\sqrt{9}+\sqrt{16}$

(13) $\sqrt{4}-\sqrt{11}$

(14) $\sqrt[3]{8}+\sqrt[3]{27}$

[2] Find an approximated value for each of the following numbers:

(1) $\sqrt{11} \cong$ ..... (to the nearest hundredth)

(2) $\sqrt[3]{7} \cong$ ..... (to the nearest tenth)

(3) $\sqrt[3]{-9} \cong$ ..... (to the nearest tenth)

[3] Find two successive integers for each the following numbers to be included between them:

(1) $\sqrt{5}$ is between ..... and .....

(2) $\sqrt{12}$ is between ..... and .....

(3) $\sqrt[3]{-20}$ is between ..... and .....

[4] If x is an integer, find the value of x in each of the following cases:

(1) $x < \sqrt{2} < x+1$ x = ...

(2) $x < \sqrt{80} < x+1$ x = ...

(3) $x < \sqrt[3]{50} < x+1$ x = ...

(4) $x < \sqrt[3]{-100} < x+1$ x = ...

(5) $x < \left| -\sqrt{35} \right| < x+1$ x = ...

[5] Complete the following:

(1) The value of $\sqrt[3]{13}$ to the nearest one decimal is .....

(2) The two consecutive integers which include the number $\sqrt{5}$ between them are ..... and .....

(3) $\sqrt[3]{x^6} = \sqrt{.....}$

(4) The solution set in Q` of the equation $5x^2 = 20$ is .....

(5) If $x \in Z$ and $x < \sqrt[3]{29} < x+1$, then x = ...

(6) If $x = \sqrt{3}$, then x^2 = ...

[6] Choose the correct answer:

(1) The irrational number in the following numbers is .....

(a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

(2) $\left(\sqrt[3]{-3}\right)^3$ = ...

(a) 3 (b) -3 (c) ± 3 (d) $\sqrt[3]{-9}$

(3) $\sqrt[3]{9}$ ..... $\sqrt{2}$

(a) < (b) > (c) = (d) $\leq$

(4) The irrational number located between 2 and 3 is .....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

(5) The irrational number between 3 and 4 is .....

(a) 3.6 (b) $\sqrt{6}$ (c) $\sqrt{15}$ (d) $\sqrt{17}$

(6) The irrational number between -2 and -1 is .....

(a) -3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

(7) $\sqrt{10} \cong$ .....

(a) 2.99 (b) 3.71 (c) 3 (d) -3.2

(8) The nearest integer to $\sqrt[3]{26}$ is .....

(a) 5 (b) 3 (c) 2 (d) 13

(9) If $n \in Z_+$, $n < \sqrt{26} < n+1$, then n = ...

(a) 25 (b) 5 (c) -5 (d) 24

(10) The area of a square whose side length is $\sqrt{3}$ cm is ..... cm^2.
(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

(11) The square whose side length is $\sqrt{7}$ cm, its area is ..... cm^2.
(a) 28 (b) 49 (c) 7 (d) 14

(12) The square whose area is 10 cm^2, its side length is ..... cm.
(a) 5 (b) -5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

(13) The S.S. of the equation $(x-\sqrt{5})(x+\sqrt{3})=0$ in Q` is .....
(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{-\sqrt{5},\sqrt{3}\}$ (d) $\{\sqrt{5},-\sqrt{3}\}$

(14) $\left|\sqrt[3]{-125}\right| = \sqrt{.....}$
(a) 25 (b) -25 (c) 5 (d) -5

[7] Find the value of x in each of the following cases and determine whether $x \in Q$ or $x \in Q'$:

(1) $5x^2 = 10$ "$\pm\sqrt{2}$"

(2) $4x^2 = 9$ "$\pm\frac{3}{2}$"

(3) $x^3 = 125$ "5"

(4) $3x^3 = 27$ "$\sqrt[3]{9}$"

(5) $0.001x^3 = -8$ "-20"

(6) $(x-1)^2 = 4$ "$3\ or\ -1$"

(7) $(x-5)^3 = 1$ "6"

[8] Find in Q` the S.S. of each of the following equations:

(1) $x^2 = 13$

(2) $x^3 = 16$

(3) $\frac{2}{5}x^2 = \frac{25}{2}$

(4) $125x^3 - 7 = 20$

(5) $\frac{1}{4}x^2 + 2 = 66$

[9] Prove that (V.I):

(1) $\sqrt{2}$ is included between 1.4 and 1.5

(2) $\sqrt{11}$ is included between 3.31 and 3.32

(3) $\sqrt{5}$ lies between 2.2 and 2.3

(4) $\sqrt[3]{15}$ lies between 2.4 and 2.5

(5) $\sqrt[3]{-17}$ lies between -2.6 and -2.5

(6) $\sqrt{3}+1$ lies between 2.7 and 2.8

Representing an irrational number on the number line

Therefore we can deduce that :

Each irrational number can be represented by a point on the number line.

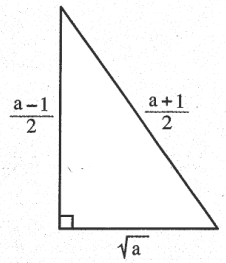
Generally

To draw a line segment with length $\sqrt{a}$ length unit where $a > 1$,

draw a right-angled triangle in which

the length of one side of the right-angle $= \frac{a-1}{2}$ length unit

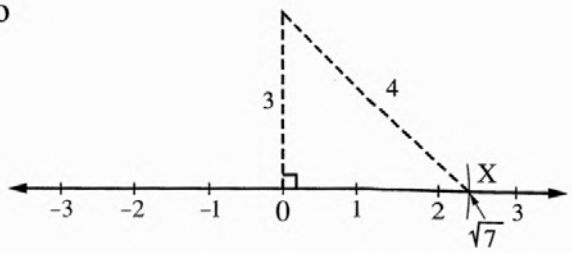
and the length of the hypotenuse $= \frac{a+1}{2}$ length unit.



Example:

Draw a line segment with length $= \sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :

Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$



[10] Determine the point that represents each of the following numbers on the number line:

(1) $\sqrt{3}$

(2) $-\sqrt{11}$

(3) $\sqrt{10}$

(4) $\sqrt{5}+1$

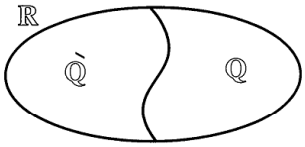
Sheet (4)

The Set of real numbers R

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

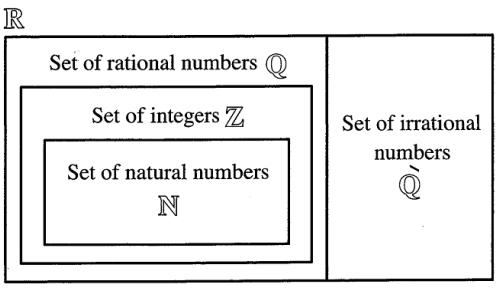


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

- **The opposite Venn diagram shows that :**

  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

  and $\grave{\mathbb{Q}} \subset \mathbb{R}$



R - Q = Q`

$R_+ = \{x : x \in R, x > 0\}$

$R_+ \cap R_- = \phi$

$\pi \in Q`$

R - Q` = Q

$R_- = \{x : x \in R, x < 0\}$

$R = R_+ \cup \{0\} \cup R_-$

$R^* = R - \{0\} = R_+ \cup R_-$

[1] Complete the following:

(1) $Q \cap Q` =$ .....

(2) $Q \cup Q` =$ .....

(3) $R_+ \cap R_- =$ .....

(4) $R_+ \cup R_- =$ .....

(5) $R - Q` =$ .....

(6) $R - Q =$ .....

(7) The solution set in R of the equation $x^2 + 9 = 0$ is .....

(8) The cube whose volume is 8 cm^3, then the sum of the lengths of its edges is ..... cm.

(9) The two integers which include the number $\sqrt{12}$ between them are ..... and .....

(10) If $\sqrt[3]{x} = -5$, then $x =$ .....

[2] Put the suitable sign (<), (>) or (=):

(1)	$\sqrt{5}$	☐	2
(2)	$\sqrt{7}$	☐	2.6
(3)	$\sqrt[3]{24}$	☐	3
(4)	$\sqrt[3]{-24}$	☐	-2
(5)	$3 - \sqrt{5}$	☐	$\sqrt[3]{-1}$
(6)	$\sqrt[3]{-8}$	☐	$\sqrt{4}$
(7)	$1 + \sqrt{3}$	☐	$\sqrt{5}$
(8)	$\sqrt[3]{3} - 1$	☐	0.2
(9)	$\sqrt{2} - 1$	☐	$1 - \sqrt{2}$

[3] Choose the correct answer from the given ones:

(1) R = ...

(a) $Q \cup Q'$ (b) $Z_+ \cup Z_-$ (c) $R_+ \cup R_-$ (d) $N \cup R_-$

(2) $\{x : x \in R, x < 0\}$ = ...

(a) R_+ (b) R_- (c) R^* (d) R

(3) If x is a negative real number, then which of the following numbers is positive?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

(4) R_+ = ...

(a) $\{x : x \in R,\ x < 0\}$ (b) $\{x : x \in R,\ x \geq 1\}$

(c) $\{x : x \in R,\ x > 0\}$ (d) $\{x : x \in R,\ x \geq 0\}$

(5) $\sqrt[3]{5}$ ..... $\sqrt{3}$

(a) < (b) > (c) = (d) $\geq$

(6) The irrational number which is included between 2 and 3 is .....

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

(7) $(-5)^{zero}$ = ...

(a) zero (b) 1 (c) -1 (d) -5

(8) The S.S. of the equation $x^2 + 1 = 0$ in R is .....

(a) $\{-1\}$ (b) $\{1, -1\}$ (c) $\{1\}$ (d) ϕ

(9) $\sqrt{(2-\pi)^2}$ ..... $(2-\pi)$

(a) < (b) > (c) = (d) $\geq$

(10) If $x \in R_+$, $y \in R_+$ and $x^2 > y^2$, then .....

(a) x > y (b) x < y (c) x = y (d) $x \leq y$

(11) If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are real numbers included between 0 and 1, then a = ...

(a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

[4] Arrange the following numbers in an ascending order:

(1) $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$

The order is: .....

(2) $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

The order is: ……

[5] Arrange the following numbers in a descending order:

(1) $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$

The order is: ……

(2) $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

The order is: ……

[6] Write three positive irrational numbers less than 2:

……

[7] Write three negative irrational numbers greater than $-\sqrt{6}$:

……

[8] Write four irrational numbers included between 15 and 17:

……

[9] Write three irrational positive numbers less than 3:

……

[10] Solve the following equations to the nearest hundredth given $x \in R$:

(1) $x^2 - 6 = 0$

(2) $\frac{3}{4}x^2 = 24$

(3) $\frac{1}{2}x^2 - 5 = 0$

(4) $5x^3 + 3 = 2$

(5) $(x^2 - 9)(x^3 - 5) = 0$

(6) $(2x^3 - 5)(x^2 + 1) = 0$

Rules to solve geometric applications

The cube

Let the edge length = ℓ,

volume = ℓ^3 , lateral area = $4\ell^2$, total area = $6\ell^2$

[11] Find the edge length of a cube whose volume is 1.728 cm^3. Is the edge length a rational number? "$\frac{6}{5}cm$"

[12] A cube whose total area is 13.5 cm^2. Find its edge length. Is the edge length a rational number? "$1.5\ cm$"

The Square

Let the edge length = S, then $area = S^2$, $S = \sqrt{area}$

Let the diagonal length = d, then $area = \frac{1}{2}d^2$, $d = \sqrt{2 \times area}$ or $d = \sqrt{2 \times s \times s}$

[13] Find the side length of a square whose area is 5 cm^2. Is the edge length a rational number? "$\sqrt{5}\,cm$"

……

……

……

[14] A square is of area 32 cm^2, Find its side length and its diagonal length? "$\sqrt{32}\,,8$"

……

……

……

[15] A square is of side length 6 cm. Find its diagonal length?"$\sqrt{72}\,cm$"

……

……

……

Sheet (1)
Medians of triangle

Definition

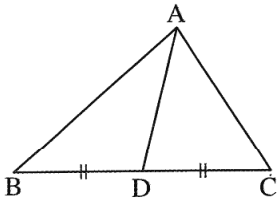
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of ΔABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

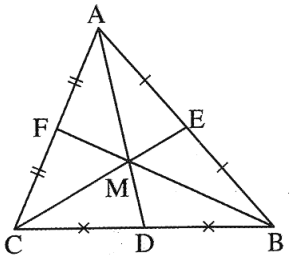
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of ΔABC ,

and they are concurrent at M

(**i.e.** $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

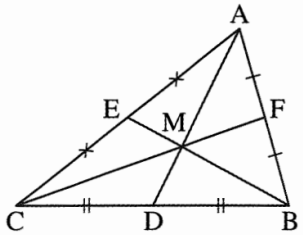
For example:

In the opposite figure :

In ΔABC , M is the point of concurrence of its medians , then :

1 $MD = \frac{1}{2} AM$ If AM = 6 cm. , then MD = 3 cm.

2 $CM = 2\ FM$ If FM = 4 cm. , then CM = 8 cm.



Remark

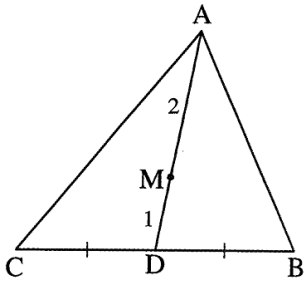
The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.

Fact

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

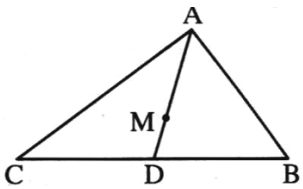
In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC and M $\in \overline{AD}$ such that AM = 2 MD ,
then M is the point of intersection of the medians of Δ ABC



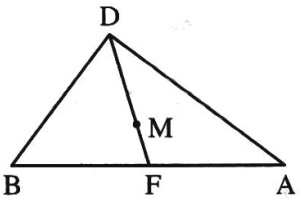
[1] Complete:

1	In Δ ABC : if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called .....
2	The medians of the triangle are .....
3	The medians of the triangle intersect at .....
4	The point of intersection of the medians of a triangle divides each median in the ratio ..... from the vertex.
5	The points of concurrence of the medians of the triangle divides each median in the ratio ..... : ..... from the base.
6	The point of intersection of the medians of the triangle divides each of them by the ratio 1 : 2 from .....
7	The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of .....
8	**In the opposite figure :** If M is the point of intersection of the medians of Δ ABC , then AM = ... AD



9 **In the opposite figure :**

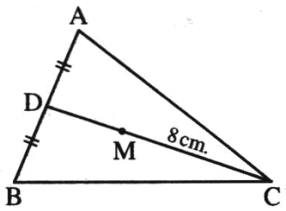
If : MF = 2 cm., then DF = ...



10 **In the opposite figure :**

In Δ ABC , M is the point of concurrence of the medians , MC = 8 cm.

, then DM = ... cm.



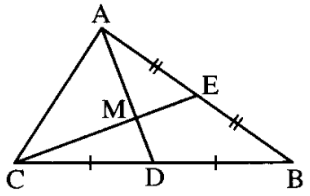
[2] Essay problems:

1 **In the opposite figure :**

E is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{BC}$

$\overline{AD} \cap \overline{CE} = \{M\}$, MC = 5 cm. and MD = 2 cm.

Find : The length of each of $\overline{AD}$ and $\overline{ME}$.

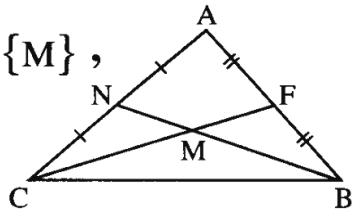


2 **In the opposite figure :**

F , N are midpoints of $\overline{AB}$, $\overline{AC}$ respectively , $\overline{BN} \cap \overline{CF} = \{M\}$,

if : AB = 8 cm. , AC = 10 cm. , BM = 4 cm. and CF = 9 cm.

Find : the perimeter of figure AFMN



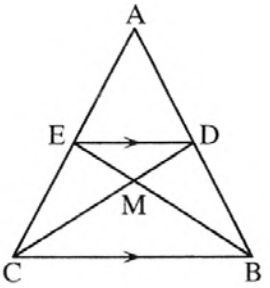
3 **In the opposite figure :**

ABC is a triangle in which $\overline{CD}$,

$\overline{BE}$ two medians intersects at M ,

if : DC = 9 cm. , BM = 4 cm. , BC = 8 cm.

Find : The perimeter of Δ MDE



Homework

[1] Choose the correct answer:

1 The medians of the triangle intersect at ..... point.
(a) 1 (b) 2 (c) 3 (d) 4

2 The right-angled triangle has ..... medians.
(a) 0 (b) 1 (c) 2 (d) 3

3 The number of medians in the right-angled triangle = ...
(a) 3 (b) 2 (c) 1 (d) 0

4 The point of intersection of the medians in the triangle divides each of them by the ratio ..... from the vertex.
(a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2

5 The point of concurrence of the medians of the triangle divides each median in the ratio of ..... from the base.
(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

6 If $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ... AD
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

7 $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then AM = ... MD
(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$

8 If $\overline{XE}$ is a median in Δ XYZ , M is the point of intersection of its medians , then EM = ... XE
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

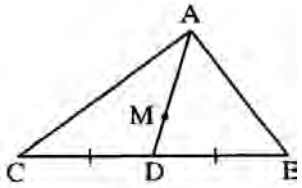
9 In Δ ABC : If AD = 6 cm. is a median and M is a point of concurrent , then MA = ... cm.
(a) 6 cm. (b) 3 cm. (c) 2 cm. (d) 4 cm.

10 If $\overline{AD}$ is a median of Δ ABC , M is the point of intersection of its medians and AM = 6 cm. , then AD = ...
(a) 12 cm. (b) 6 cm. (c) 18 cm. (d) 9 cm.

11 **In the opposite figure :**

$\overline{AD}$ is a median in ΔABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = ... cm.

(a) 2 (b) 4 (c) 6 (d) 8

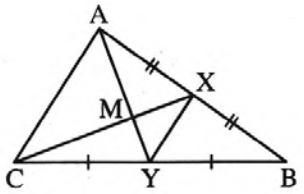


[2] Essay problems:

1 **In the opposite figure :**

ABC is a triangle , X bisects $\overline{AB}$, Y bisects $\overline{BC}$, XY = 5 cm. , $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm. , YM = 3 cm.

Find with proof the length of : $\overline{AC}$, $\overline{MX}$, $\overline{AM}$

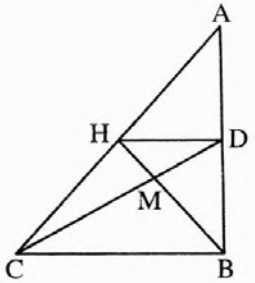


2 **In the opposite figure :**

ABC is a triangle in which $\overline{CD}$, $\overline{BH}$ are medians intersect at M ,

MC = 6 cm. , BC = 8 cm. , MB = 4 cm.

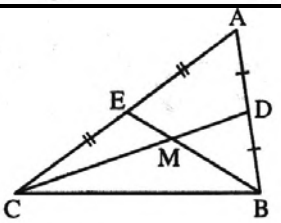
Find with proof : The perimeter of ΔMDH



3 **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overline{BE} \cap \overline{CD} = \{M\}$, If AB = 6 cm. , AC = 10 cm. , BM = 4 cm. and CD = 9 cm.

Find the perimeter of the figure : ADME



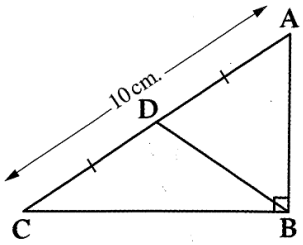
Sheet (2)
Medians of triangle (Follow)

Theorem 3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

For example:

In the opposite figure :



Δ ABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. ,

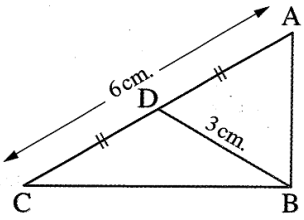
then DB = 5 cm.

The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

For example:

In the opposite figure :



If $\overline{BD}$ is a median in Δ ABC ,

BD = 3 cm. and AC = 6 cm. ,

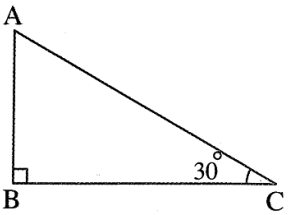
then m (∠ ABC) = 90° "because BD = $\frac{1}{2}$ AC "

Corollary

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure :



If Δ ABC is right-angled at B and

m (∠ C) = 30° , then AB = $\frac{1}{2}$ AC

For example:

If AC = 20 cm. , then AB = 10 cm.

Remark

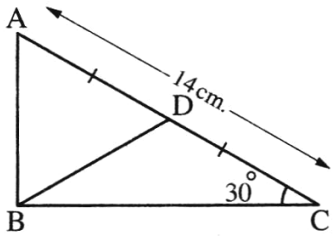
The right-angled triangle whose measures of angles are 30° , 60° and 90° is called thirty and sixty triangle.

[1] Complete:

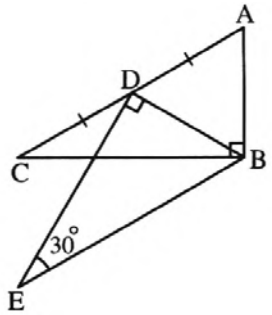
1	In the right-angled triangle the length of the median from the vertex of the right angle equal ..... the length of the hypotenuse.
2	In the right-angled triangle , the length of the median from the vertex of the right angle equals .....
3	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then .....
4	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ..... the length of the hypotenuse.
5	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ...
6	The length of the hypotenuse on the right-angled triangle equals ..... the length of a side opposite to the angle of measure 30°
7	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ... cm.
8	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ... cm.
9	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ... cm.
10	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ... AC
11	If ABC : Is a right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ C) = ...
12	If ABC is a right-angled triangle at B and AB = $\frac{1}{2}$ AC , then m (∠ A) = ...

13 ABC is a right-angled triangle at B , if AC = 2 BC , then m (∠ C) = ...°

14 In the opposite figure :
The perimeter of Δ ABD = ... cm.

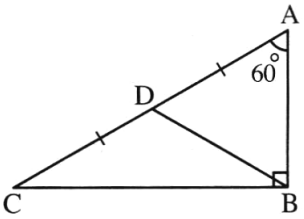


15 **In the opposite figure :**
D is the midpoint of $\overline{AC}$
, m (∠ E) = 30°
, AC = 10 cm.
Find the length of : $\overline{BE}$

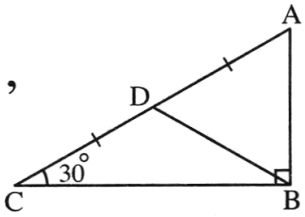


[2] Essay problems:

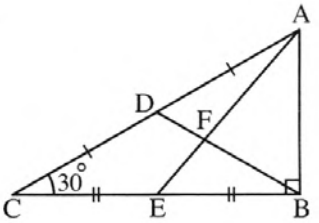
1 **In the opposite figure :** Δ ABC , AC = 8 cm. ,
m (∠ BAC) = 60° , m (∠ ABC) = 90° ,
D is the midpoint of $\overline{AC}$
Find : The perimeter of Δ ABD



2 **In the opposite figure :**
m (∠ B) = 90° , m (∠ C) = 30° , $\overline{BD}$ is a median , AB = 4 cm. ,
Complete :
AC = ... cm. , BD = ... cm. , AD = ... cm.



3 **In the opposite figure :**
Δ ABC in which m (∠ B) = 90° , AC = 10 cm. ,
m (∠ C) = 30° , EC = EB , AD = DC
Find with proof : ❶ The perimeter of Δ ABD
❷ The length of $\overline{DF}$



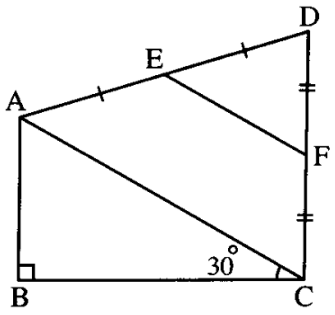
4 **In the opposite figure :**

$m(\angle B) = 90°$,

$m(\angle ACB) = 30°$,

E , F are midpoints of $\overline{AD}$, $\overline{DC}$

Prove that : AB = EF



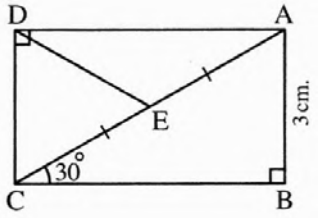
5 **In the opposite figure :**

$m(\angle ABC) = m(\angle ADC) = 90°$,

$m(\angle ACB) = 30°$, and $\overline{DE}$ is a median of ΔADC ,

If AB = 3 cm.

Find : The length of $\overline{DE}$



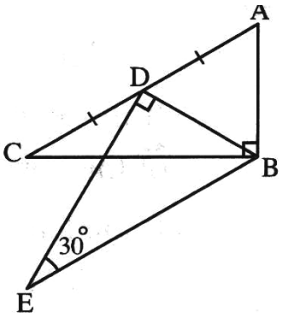
6 **In the opposite figure :**

$m(\angle ABC) = m(\angle BDE) = 90°$

, $m(\angle E) = 30°$

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



Homework

[1] Choose the correct answer:

1 The length of the hypotenous of the right-angled triangle = ... the length of the median which drawn from the vertex of the right-angle.

(a) half (b) twice (c) third (d) quarter

2 The length of the median drawn from the vertex of right angle in the right-angled triangle = ... the length of the hypotenuse of the triangle.

(a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3 In the right-angled triangle , the length of the median from the vertex of the right angle equals ..... the length of hypotenuse.

(a) half (b) twice (c) third (d) forth

4 If Δ ABC is a right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the medians drawn from B is ..... cm.

(a) 10 (b) 8 (c) 6 (d) 5

5 In Δ ABC which is right at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals .....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

6 In Δ ABC , m ($\angle$ B) = 90° , AC = 12 cm. and $\overline{BD}$ is a median in Δ ABC , then BD = ... cm.

(a) 12 (b) 6 (c) 24 (d) 10

7 The length of the side opposite to the angle of measure 30° in the right-angled ..... the length of the hypotenuse.

(a) twice (b) half (c) square (d) equals

8 Triangle ABC : If m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , then BC = ...

(a) $\frac{1}{2}$ AB (b) $\frac{1}{2}$ AC (c) 2 AB (d) 2 AC

9 In Δ ABC if : m ($\angle$ B) = 90° and m ($\angle$ A) = 60° , then AC = ... AB

(a) 2 (b) = (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

10 Δ ABC : if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then AC = ...

(a) $\frac{1}{2}$ BC (b) 2 BC (c) 2 AB (d) BC

11 In Δ ABC : m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , AC = 10 cm. , then BC = ... cm.

(a) 20 (b) 15 (c) 10 (d) 5

12 In Δ XYZ , if m ($\angle$ Y) = 90° , m ($\angle$ X) = 30° and XZ = 20 cm. , then ZY = ... cm.

(a) 5 (b) 8 (c) 20 (d) 10

13 In the rectangle ACBD , if AC = 10 cm. , then BD = ...

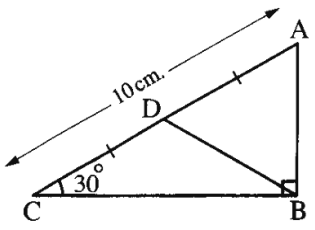
(a) 5 (b) 10 (c) 15 (d) 20

[2] Complete:

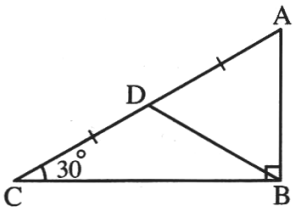
1	In the right-angled triangle , the length of the median from the vertex of the right angle equals .....
2	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then .....
3	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ..... the length of the hypotenuse.
4	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ...
5	The length of the hypotenuse on the right-angled triangle equals ..... the length of a side opposite to the angle of measure 30°
6	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ... cm.
7	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ... cm.
8	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ... cm.
9	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ... AC

[3] Essay problems:

1	**In the opposite figure :** m (∠ B) = 90° and m (∠ C) = 30° , AC = 10 cm. **Find :** the lengths of $\overline{AB}$ and $\overline{BD}$



2	**In the opposite figure :** m (∠ C) = 30° **Prove that :** AB = BD



3 **In the opposite figure :**

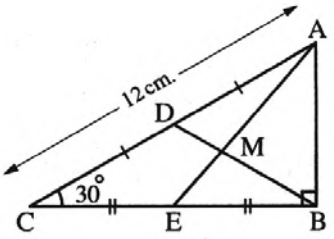
In Δ ABC : m ($\angle$ B) = 90° , m ($\angle$ C) = 30°

, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

, AC = 12 cm.

(1) **Find length of :** $\overline{BM}$

(2) **Find the perimeter of :** Δ ABC



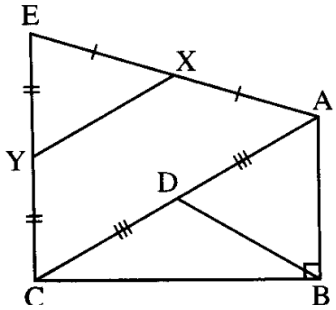
4 **In the opposite figure :**

X , Y , D are the midpoints of

$\overline{EA}$, $\overline{EC}$ and $\overline{AC}$ respectively ,

m ($\angle$ ABC) = 90°

Prove that : BD = YX

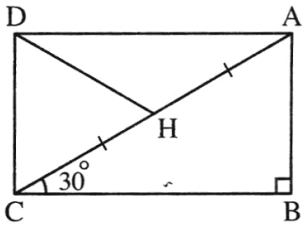


5 **In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30° ,

AB = DH where H is midpoint of $\overline{AC}$

Prove that : m ($\angle$ ADC) = 90°



6 **In the opposite figure :**

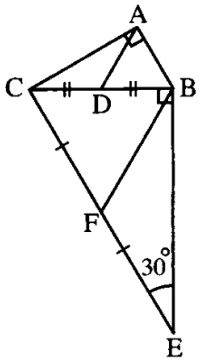
m ($\angle$ BAC) = m ($\angle$ CBE) = 90° ,

m ($\angle$ BEC) = 30° ,

D and F are the midpoints of $\overline{BC}$

and $\overline{CE}$ respectively.

Prove that : AD = $\frac{1}{2}$ BF

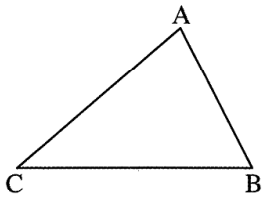


Sheet (3)

The isosceles triangle

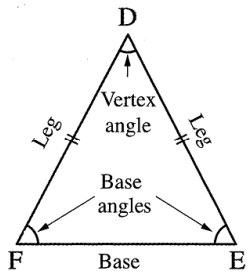
Triangles are classified according to the lengths of their sides into three types which are :

1 Scalene triangle.



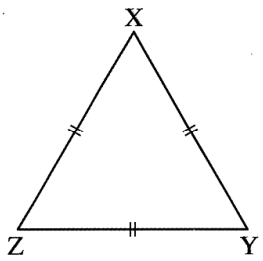
$AB \neq BC \neq CA$

2 Isosceles triangle.
(two sides are congruent).



$DE = DF$

3 Equilateral triangle.
(three sides are congruent).



$XY = YZ = ZX$

And in the following we will study the relations between the angles in the isosceles triangle and the equilateral triangle.

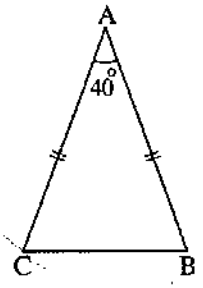
The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.

For example:

In the opposite figure :



If ABC is a triangle in which :

$AB = AC$, $m(\angle A) = 40°$,

then $m(\angle B) = m(\angle C) = \frac{180° - 40°}{2} = 70°$

Remarks

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

Corollary

If the triangle is equilateral , then it is equiangular where each angle measure is 60°

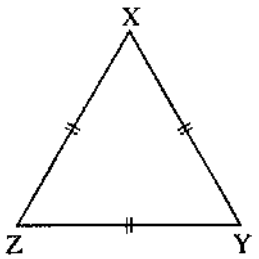
For example:

In the opposite figure :

If XYZ is a triangle in which

XY = YZ = ZX ,

then m (∠ X) = m (∠ Y) = m (∠ Z) = 60°



[1] Complete:

1	The two base angles in an isosceles triangle are .....
2	Δ ABC , AB = AC , m (∠ C) = 70° , then m (∠ A) = ...
3	In the Δ ABC : AB = AC , m (∠ A) = 70° , then m (∠ C) = ...°
4	The Δ ABC is an isosceles and right-angled triangle if m (∠ B) = 90° , then m (∠ A) = m (∠ C) = ...°
5	In Δ ABC , if AB = AC and m (∠ A) = 80° , then m (∠ B) = m (∠ .....) = ...°
6	In Δ ABC : if AB = AC , m (∠ B) = 60° , then the triangle is an .....
7	In Δ ABC : If AB = AC and m (∠ A) = 2 m (∠ C) , then m (∠ B) = ... °
8	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ...
9	The length of the hypotenuse on the right-angled triangle equals ..... the length of a side opposite to the angle of measure 30°
10	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ... cm.
11	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ... cm.
12	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ... cm.

[2] Essay problems:

1

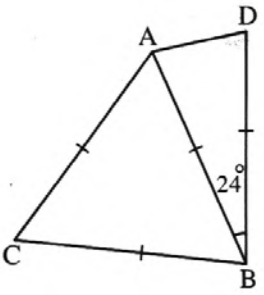
In the opposite figure :

ACBD is a quadrilateral in which :

AB = BC = CA = BD

, m (∠ ABD) = 24°

Find : m (∠ CAD)



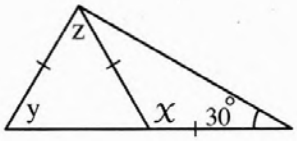
2

In the opposite figure complete :

x = ... ° ,

y = ... ° ,

z = ... °



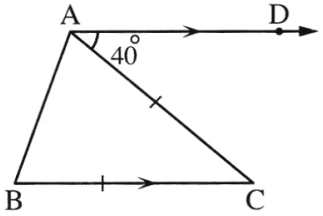
3

In the opposite figure :

ABC is a triangle ,

AC = BC , $\overrightarrow{AD}$ // $\overline{BC}$, m (∠ DAC) = 40°

Find : The measure of angles in the Δ ABC



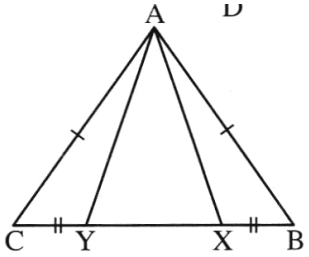
4

In the opposite figure :

In Δ ABC , AB = AC ,

BX = CY

Prove that : AX = AY



Homework

[1] Choose the correct answer:

1

In any isosceles triangle , the type of the base angles is .....

(a) acute. (b) right. (c) obtuse. (d) reflex.

2

The base angles of the isosceles triangle are .....

(a) congruent. (b) alternate. (c) corresponding. (d) supplementary.

3

In Δ ABC : AB = AC , m (∠ B) = 50° , then m (∠ A) = ...°

(a) 65 (b) 80 (c) 50 (d) 100

4 If measure of one of the two base angles of the isosceles triangle equals 40° then the measure of the vertex angle = ...°

(a) 40 (b) 100 (c) 80 (d) 50

5 An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle = ...

(a) 50° (b) 60° (c) 70° (d) 80°

6 In the isosceles triangle , if the measure of one of the two base angle is 70° , then the measure of its vertex angle is .....

(a) 70° (b) 110° (c) 20° (d) 40°

7 The measure of one angle of the two base angles of the isosceles = 75° , then the measure of the vertex angle = ...

(a) 50° (b) 75° (c) 30° (d) 105°

8 In a triangle ABC : If AB = AC and m (∠ A) = 40° , then m (∠ C) = ...

(a) 40° (b) 70° (c) 140° (d) 50°

9 In Δ ABC , AB = AC , m (∠ A) = 50° , then m (∠ B) = ...

(a) 50° (b) 65° (c) 130° (d) 100°

10 If the measure of an angle of the isosceles triangle is 100° , then the measure of one of the other angles = ...

(a) 50° (b) 80° (c) 40° (d) 100°

11 If the measure of an angle of the isosceles triangles is 120° , then the measure of one of the other angles = ...

(a) 60° (b) 30° (c) 40° (d) 45°

12 The triangle whose sides lengths are 2 cm. , $(x + 1)$ cm and 5 cm. becomes an isosceles triangle when x = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

13 Triangle whose sides lengths are 2 cm. , $(x - 2)$ cm. , 5 cm. becomes isosceles triangle when x = ... cm.

(a) 3 (b) 4 (c) 5 (d) 7

14 ABC is a triangle in which AB = AC and m (∠ A) = 110° , then m (∠ B) = ...

(a) 70° (b) 55° (c) 35° (d) 110°

15 Δ XYZ is an isosceles triangle in which m (∠ X) = 100° , then m (∠ Y) = ...°

(a) 100 (b) 80 (c) 60 (d) 40

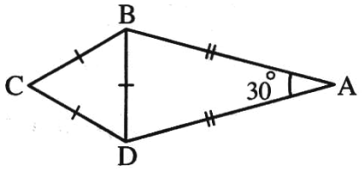
[2] Essay problems:

1 **In the opposite figure :**

AB = AD , m (∠ A) = 30° ,

CB = BD = CD

Find : m (∠ CBA)

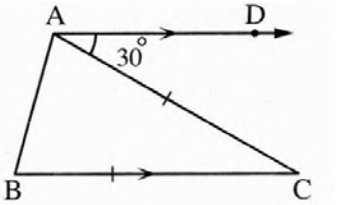


2 **In the opposite figure :**

ABC is a triangle in which : AC = BC ,

$\overline{AD}$ // $\overline{BC}$, m (∠ DAC) = 30°

Find : m (∠ ABC)

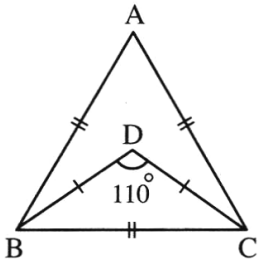


3 **In the opposite figure :**

ABC is an equilateral triangle ,

DB = DC , m (∠ D) = 110°

Find with proof : m (∠ DBC) and m (∠ DBA)

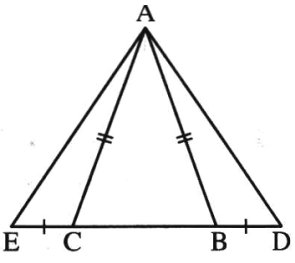


4 **In the opposite figure :**

ADE is a triangle , B ∈ $\overline{DE}$, C ∈ $\overline{DE}$

, BD = CE , AB = AC

Prove that : AD = AE



Sheet (4)

The converse of the isosceles triangle theorem

Theorem 2

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Remark

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

[1] Complete:

1	If angles of any triangle are equal in measures , then the triangle is .....
2	If the angles of a triangle are congruent , then the triangle is .....
3	The measure of the exterior angle of equilateral triangle = ...°
4	If the measure of one of the angles of the right-angled triangle is 45° , then the triangle is .....
5	In an isosceles triangle , if any angle has a measure of 60° , the triangle is .....
6	In Δ ABC if : $\overline{AB} \perp \overline{BC}$ and AB = BC , then m (∠ A) = ...°

[2] Essay problems:

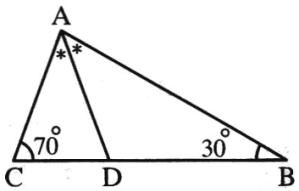
1 **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC

, m (∠ B) = 30°

, m (∠ C) = 70°

Prove that : Δ ADC is isosceles triangle.



2 ABC is a triangle in which : m (∠ A) = 50° and m (∠ C) = 80°

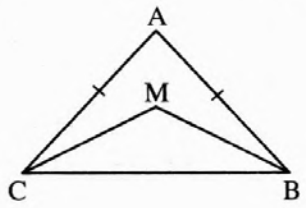
Prove that : this triangle ABC is an isosceles triangle.

3 **In the opposite figure :**

AB = AC ,

$\overrightarrow{BM}$ and $\overrightarrow{CM}$ bisect the angles (∠ B) , (∠ C)

Prove that : MB = MC



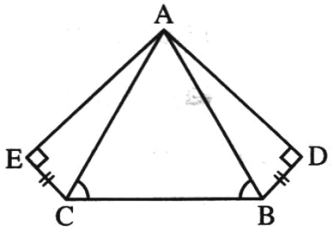
4 **In the opposite figure :**

BD = CE

, m (∠ ABC) = m (∠ ACB)

, m (∠ D) = m (∠ E) = 90°

Prove that : m (∠ DAB) = m (∠ CAE)

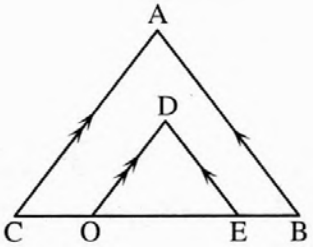


5 **In the opposite figure :**

AB = AC , $\overline{DE}$ // $\overline{AB}$

and $\overline{AC}$ // $\overline{DO}$

Prove that : (1) DE = DO (2) m (∠ A) = m (∠ D)



Homework

[1] Choose the correct answer:

1 The measure of exterior angle of an equilateral triangle = ...

(a) 30° (b) 60° (c) 120° (d) 180°

2 In Δ XYZ : if XY = XZ , then the exterior angle at the vertex Z is .....

(a) acute. (b) obtuse. (c) right. (d) reflex.

3 In Δ ABC : if AB = AC and m (∠ A) = 60° , if its perimeter is 18 cm. , then BC = ... cm.

(a) 18 (b) 6 (c) 3 (d) 60

[2] Essay problems:

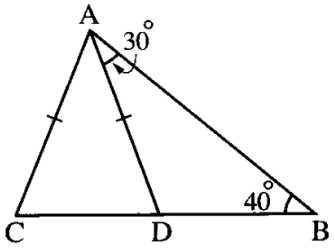
1 **In the opposite figure :**

AD = AC

, m (∠ DAB) = 30°

, m (∠ ABD) = 40°

Prove that : AB = CB

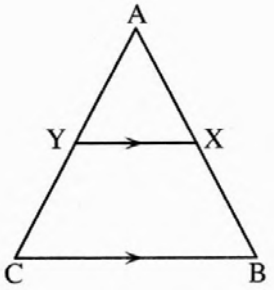


2 **In the opposite figure :**

ABC is a triangle in which AB = AC , X ∈ $\overline{AB}$,

Y ∈ $\overline{AC}$ and $\overline{XY}$ // $\overline{BC}$

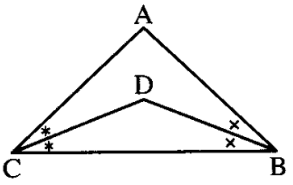
Prove that : the triangle AXY is isosceles triangle.



3 **In the opposite figure :**

AB = AC , $\overrightarrow{BD}$ bisects ∠ B and $\overrightarrow{CD}$ bisects ∠ C

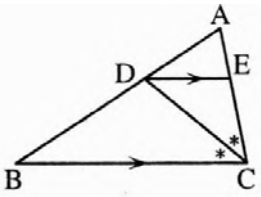
Prove that : Δ DBC is an isosceles triangle



4 **In the opposite figure :**

$\overrightarrow{CD}$ bisects ∠ ACB , $\overline{DE}$ // $\overline{CB}$

Prove that : Δ ECD is an isosceles triangle.



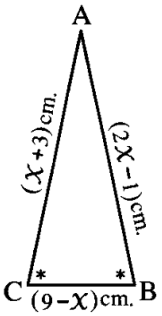
5 **In the opposite figure :**

m (∠ B) = m (∠ C) , AB = (2 $\mathcal{X}$ – 1) cm.

AC = ($\mathcal{X}$ + 3) cm.

, BC = (9 – $\mathcal{X}$) cm.

Find with proof the perimeter of Δ ABC



Test 1

Total mark 10

1 Choose the correct answer from the given ones : (3 marks)

1 If $-\sqrt{25} = \sqrt[3]{y}$, then y = ...

ⓐ 5 ⓑ –5 ⓒ 125 ⓓ –125

2 The irrational number included between –2 and –1 is .....

ⓐ –3 ⓑ $-1\frac{1}{3}$ ⓒ $-\sqrt{3}$ ⓓ $\sqrt{2}$

3 If x is a negative real number , then which of the following represents a positive number ?

ⓐ x^2 ⓑ x^3 ⓒ $3x$ ⓓ $\frac{x}{3}$

2 Complete : (3 marks)

1 $\mathbb{R}_+ \cup \mathbb{R}_-$ = ...

2 The S.S. of the equation : $(x-\sqrt{5})(x+\sqrt{3}) = 0$ in $\mathbb{Q}$ is .....

3 A square of area 7 cm^2 , then its side length = ... cm.

3 Prove that : $\sqrt{2}$ lies between 1.4 and 1.5 (2 marks)

4 The capacity of a cube is 27 litres. Find its inner edge length. (2 marks)

Test 2

Total mark 10

1 Choose the correct answer from the given once : (3 marks)

1 $\mathbb{R}_+ \cap \mathbb{R}_- =$ .....

(a) $\mathbb{R}^*$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\varnothing$

2 $\sqrt[3]{0.001 \times \frac{1}{8}} =$ .....

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

3 A square of side length $\sqrt{3}$ cm. , then its area is ..... cm^2

(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

2 Complete : (3 marks)

1 If $x^3 = 27$, then $x =$ .....

2 $\mathbb{Q} \cup \hat{\mathbb{Q}} =$ .....

3 The S.S. of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is .....

3 Find in $\mathbb{R}$ the S.S. of the equation : $2 + x^3 = 1$ (2 marks)

4 Find the value of x in each of the following : (2 marks)

1 $\sqrt[3]{x} = \frac{1}{2}$

2 $x^3 + 5 = 32$

Test 1

Total mark 10

1 Choose the correct answer from the given ones : (3 marks)

1 The number of medians of the right-angled triangle is .....

(a) zero (b) 1 (c) 2 (d) 3

2 ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, then BD = ...

(a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB

3 Δ XYZ is an isosceles triangle in which , m ($\angle$ Y) = 100° , then m ($\angle$ Z) = ...

(a) 100° (b) 80° (c) 50° (d) 40°

2 Complete : (3 marks)

1 The length of the hypotenuse in the right-angled triangle equals ..... the length of the median drawn from the vertex of the right angle.

2 The measure of the exterior angle of the equilateral triangle equals .....°

3 The point of intersection of medians of the triangle divides each of them in the ratio ..... : 2 from the base.

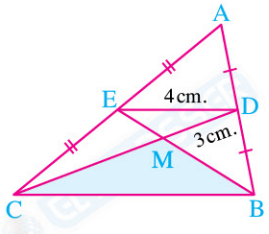
3 In the opposite figure : (2 marks)

If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm.

, DM = 3 cm. , BE = 6 cm.

Find : The perimeter of Δ BMC

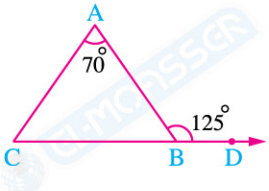


4 In the opposite figure : (2 marks)

D $\in \overrightarrow{CB}$, m ($\angle$ ABD) = 125°

and m ($\angle$ A) = 70°

Prove that : Δ ABC is an isosceles triangle.



Test 2

Total mark 10

1 Choose the correct answer from the given ones : (3 marks)

1 If M is the point of concurrence of medians of Δ ABC , $\overline{BD}$ is a median , then BM : MD = ...

ⓐ 2 : 3 ⓑ 2 : 1 ⓒ 3 : 1 ⓓ 1 : 2

2 In Δ ABC , if m (∠ B) = 90° and m (∠ C) = 30° , then AB = ... AC

ⓐ $\frac{1}{2}$ ⓑ $\frac{1}{3}$ ⓒ twice ⓓ $\frac{1}{4}$

3 If the measure of one of the base angles of an isosceles triangle is 45° , then the triangle is ..... triangle.

ⓐ obtuse-angled. ⓑ acute-angled. ⓒ right-angled. ⓓ equilateral.

2 Complete : (3 marks)

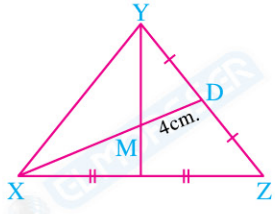
1 The medians of the triangle intersect at .....

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .....

3 **In the opposite figure :**

If DM = 4 cm.

, then XD = ... cm.

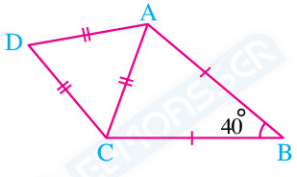


3 In the opposite figure : (2 marks)

AD = DC = AC , AB = BC

, m (∠ ABC) = 40°

Find : m (∠ BAD)



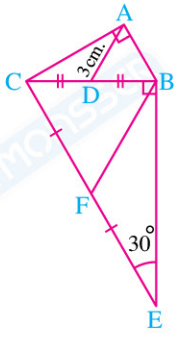
4 In the opposite figure : (2 marks)

m (∠ BAC) = m (∠ CBE) = 90° , m (∠ BEC) = 30°

, D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively

, AD = 3 cm.

Find : the length of $\overline{BF}$



Answers of Test 1

1 1 (d) 2 (c) 3 (a)

2 1 $\mathbb{R}^*$ or $\mathbb{R} - \{0\}$ 2 $\{\sqrt{5}, -\sqrt{3}\}$ 3 $\sqrt{7}$

3 $\because (\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$, $(1.4)^2 = 1.96$, $(1.5)^2 = 2.25$

$\therefore 1.96 < 2 < 2.25$ $\therefore \sqrt{1.96} < \sqrt{2} < \sqrt{2.25}$

$\therefore 1.4 < \sqrt{2} < 1.5$ $\therefore \sqrt{2}$ lies between 1.4 , 1.5

4 27 litres $\times$ 1000 = 27000 cm^3 $\because$ volume of the cube = ℓ^3

$\therefore \ell^3 = 27000$ $\therefore \ell = \sqrt[3]{27000}$ $\therefore \ell = 30$ cm.

Answers of Test 2

1 1 (d) 2 (c) 3 (c)

2 1 3 2 $\mathbb{R}$ 3 $\varnothing$

3 $\because 2 + x^3 = 1$ $\therefore x^3 = 1 - 2 = -1$

$\therefore x = \sqrt[3]{-1} = -1$ $\therefore$ The S.S. = $\{-1\}$

4 1 $\because \sqrt[3]{x} = \frac{1}{2}$ $\therefore x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

2 $\because x^3 + 5 = 32$ $\therefore x^3 = 32 - 5 = 27$

$\therefore x = \sqrt[3]{27} = 3$

Answers of Test 1

1 1 (d) 2 (a) 3 (d)

2 1 twice 2 120° 3 1

3 $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$ (given)

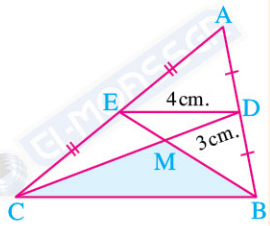
$\therefore$ BC = 2DE $\therefore$ BC = 8 cm.

$\because$ M is the point of intersection of medians of Δ ABC

$\therefore$ MC = 2DM $\therefore$ MC = 6 cm.

, $BM = \frac{2}{3}$ BE $\therefore$ BM = 4 cm.

$\therefore$ The perimeter of Δ BMC = 8 + 6 + 4 = 18 cm. (The req.)



4 $\because$ B $\in \overline{DC}$

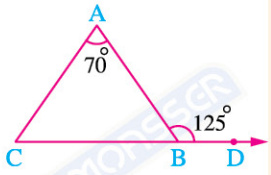
$\therefore$ m ($\angle$ ABC) = $180^\circ - 125^\circ = 55^\circ$

In Δ ABC : m ($\angle$ C) = $180^\circ - (55^\circ + 70^\circ) = 55^\circ$

$\therefore$ m ($\angle$ ABC) = m ($\angle$ C)

$\therefore$ AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (The req.)

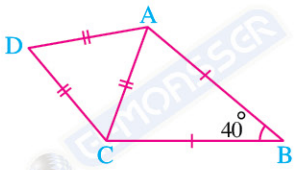


Answers of Test 2

1 1 (b) 2 (a) 3 (c)

2 1 one point 2 half length of the hypotenuse 3 12 cm.

3 ∵ Δ ACD is an equilateral triangle



∴ m (∠ CAD) = 60° (1)

In Δ ABC :

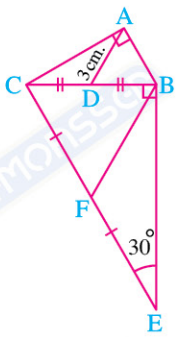
∵ AB = BC

∴ $m(\angle BAC) = m(\angle BCA) = \frac{180° - 40°}{2} = 70°$ (2)

From (1) , (2) :

∴ m (∠ BAD) = 60° + 70° = 130° (The req.)

4 In Δ ABC :



∵ m (∠ BAC) = 90° , D is the midpoint of $\overline{BC}$

∴ BC = 2AD = 2 × 3 = 6 cm.

In Δ CBE :

∵ m (∠ CBE) = 90° , m (∠ E) = 30°

∴ EC = 2BC = 2 × 6 = 12 cm.

∵ F is the midpoint of $\overline{EC}$

∴ $BF = \frac{1}{2} EC = \frac{1}{2} \times 12 = 6$ cm. (The req.)

Revision sheet of November month

Algebra:

Remark :-

$$R = R^+ \cup \{0\} \cup R^-$$

Remark : The set of non-negative real numbers = $R^+ \cup \{0\} = \{x : x \geqslant 0, x \in R\}$

The set of the non - positive real numbers = $R^- \cup \{0\} = \{x : x \leqslant 0, x \in R\}$

$$Q \cup Q' = R$$

(1) Complete:

1) $Q \cup Q' = ...$

2) $-\sqrt{25} = \sqrt[3]{X}$,thenX=.....

3)if $x \in z, x < \sqrt{11} < x+1$,*then* $x = ...$

4) $\sqrt{7} \in [x, x+1], x \in z$ then x=.....

5) the two consective integers which included the number $\sqrt{5}$ *are.....and.....*

6) the S.S of $8x^3 + 1 = -7$ in Ris.....

7)If $\sqrt[3]{x} = \sqrt{9}$, then x=.....

8) $\sqrt[3]{x^6} = \sqrt{.....}$

9) The set of non- negative real number=.....(Write as intervals)

10) $R^+ = ...$ (Write as intervals)

11) the set of non positive real numbers =.....

12) s.s of $x^2 + 9 = 0$ in R *is*.....

13)*s.s of* $x^3 - 2 = 3$ in R *is*.....

14) $\left|\sqrt[3]{-27}\right| = \sqrt{.....}$

15)*s.s of* $x(x-2) = 0$ in R *is*.....

16)*s.s of* $(x+3)(x-2) = 0$ in R *is*.....

17)if $x \in z, x < -\sqrt{11} < x+1$,*then* $x = ...$

18) the sum of the square roots of the number 25 =.....

(2) Choose the correct answer from those given :

1 $\mathbb{R}$ = ...

(a) $\mathbb{Q} \cup \grave{\mathbb{Q}}$ (b) $\mathbb{Z}_+ \cup \mathbb{Z}_-$ (c) $\mathbb{R}_+ \cup \mathbb{R}_-$ (d) $\mathbb{N} \cup \mathbb{R}_-$

2 $\mathbb{Q} \cap \grave{\mathbb{Q}}$ = ...

(a) $\mathbb{Q}$ (b) $\grave{\mathbb{Q}}$ (c) $\mathbb{R}$ (d) $\varnothing$

3 $\mathbb{Q} \cup \grave{\mathbb{Q}}$ = ...

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\grave{\mathbb{Q}}$

4 $\mathbb{R}_+ \cap \mathbb{R}_-$ = ...

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}_-$

5 $\mathbb{R}_+ \cup \mathbb{R}_-$ = ...

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}^*$

6 $\mathbb{R} - \grave{\mathbb{Q}}$ = ...

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\{0\}$

7 $\mathbb{R} - \mathbb{Q}$ = ...

(a) $\grave{\mathbb{Q}}$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\{0\}$

8 $\mathbb{R}_+ \cap \{-1, 0, 1\}$ = ...

(a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$

9 $\{x : x \in \mathbb{R}, x < 0\}$ = ...

(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

10 If x is a negative real number , then which of the following numbers is positive ?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

(3) **Prove :** $\sqrt{11}$ lies between 3 and 4

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(4) **Prove :** $\sqrt[3]{15}$ lies between 2.4 and 2.5

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(5) Draw the number line and represent $1+\sqrt{7}$ on the number

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(6)Find the S.S of each in Q the following:-

(1) $2X^2 - 3 = 47$

(2) $X^3 - \frac{3}{8} = 3$

(3) $(X^3 - 1)(X^2 - 4) = 0$

(4) $(X+1)^3 = 125$

(7)Find the S.S of each in R the following:-

(1) $2X^3 + 4 = 16$

(2) $\frac{1}{8}X^3 - 3 = 5$

(3) $(X+1)^2 = 25$

(4) $(5X-2)^3 + 2 = 10$

Geometry:

(1): Complete::

1 a) In Δ ABC if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called .....

b) The medians of the triangle intersect at .....

c) The point of intersection of the medians of the triangle divides each of them in the ratio of ..... : ..... from the base.

d) The points which divides the median of the triangle in the ratio 1 : 2 from the base is the point of .....

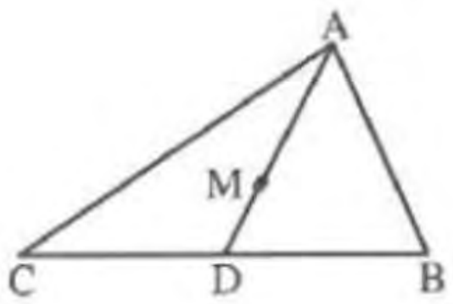
e) In the opposite figure :

If M is the point of intersection of the medians of Δ ABC then :

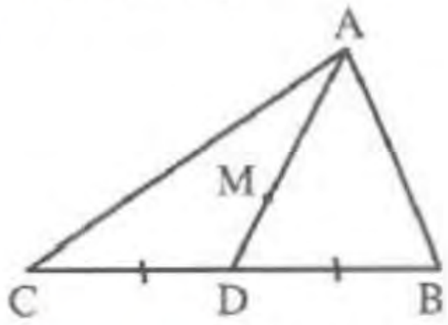
First : BD = ... BC

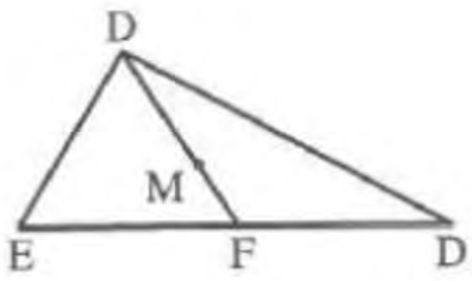
Second : AM = ... MD

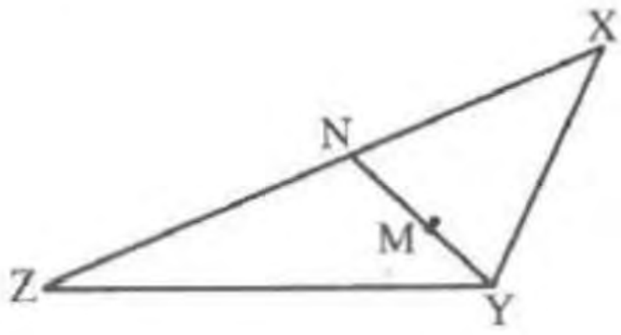
Third : AM = ... AD



2 In each of the following figures M is the point of intersection of the medians of the given triangle.







a) Fig. (1) : If AM = 2 cm. then MD = ... cm.

a) Fig. (2) : If MF = 1.5 cm. then DF = ... cm.

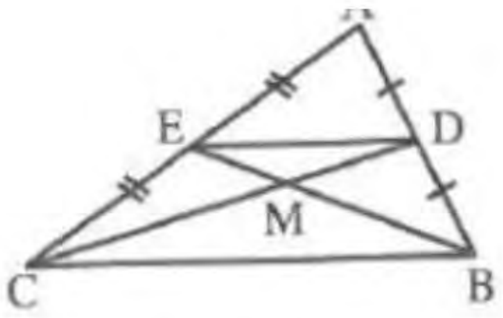
a) Fig. (3) : If YN = 6 cm. then YM = ... cm.

3 *In the opposite figure :*

a) If : DE = 3 cm. , then BC = ... cm.

b) If : CD = 4.5 cm. then CM = ... cm.

c) If : ME 2 2 cm. then BE = ... cm.



4 a) The length of the median of the right angled triangle which is drawn from the vertex of the right angle equals .....

b) If the length of the median of the triangle which is drawn from one of its vertice equal half the length of the opposite side to this vertex , then .....

c) The length of the side opposite to the angle of measure 30° in the rigth angled triangle equal .....

5 *In each of the following figures :*

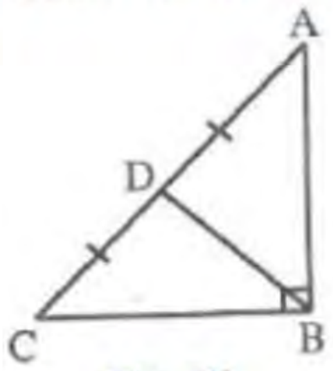


Fig. (1)

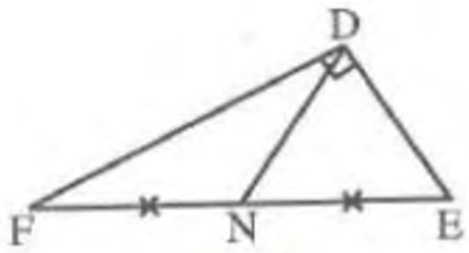


Fig. (2)

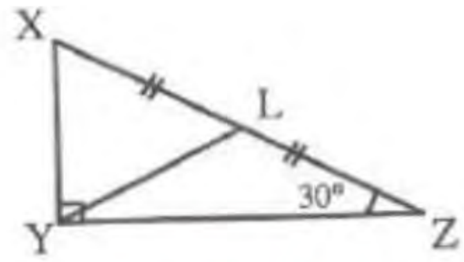


Fig. (3)

a) In Fig. (1) : : If AC = 8 cm. then BD = ... cm.

b) In Fig. (2) : : If DN = 3 cm. then EN = ... cm.

c) In Fig. (3) : : If XY = 3.5 cm. then YL = ... cm.

6 *In the opposite figure :*

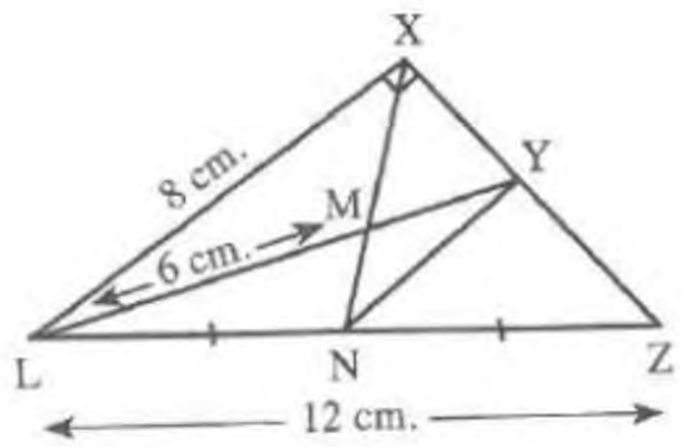
$\overline{XN}$ and $\overline{YL}$ are two medians

m (∠ ZXL) = 90° , ZL = 12 cm.

XL = 8 cm. , ML = 6 cm.

a) XN = ... cm. b) YN = ... cm.

b) MY = ... cm. c) YL = ... cm.



(2) Choose the correct answer from those given:

1. If M is the point of intersection of the medians of Δ ABC and D is the midpoint of $\overline{BC}$, then AD = ...

(a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

2. The point of intersection of the medians of the triangle divides each of them with the ratio ..... from the vertex.

(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 3 : 2

3. If M is the point of intersections of the medians of the triangle in Δ ABC and $\overline{AX}$ is a median of length 6 cm. then AM equals .....

(a) 1 (b) 2 cm. (c) 3 cm. (d) 4 cm.

4. ABCD is a rectangle M is the point of intersection of its diagonals. If the length of the diagonal is 6 cm. then the length of the median $\overline{AM}$ equals .....

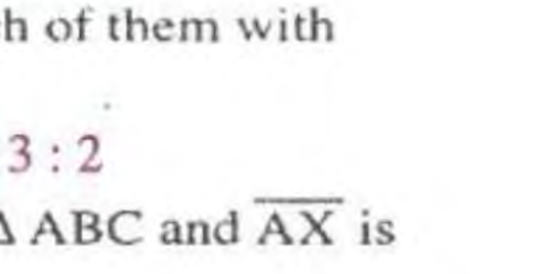
(a) 2 cm. (b) 3 cm. (c) 6 cm. (d) 12 cm.

(3) Answer each the following with prove:

1 *In the opposite figure :*

m (∠ ABC) = 90° , D is the midpoint of $\overline{AC}$, m (∠ C) = 30° . Prove that :

Δ ABD is equilateral



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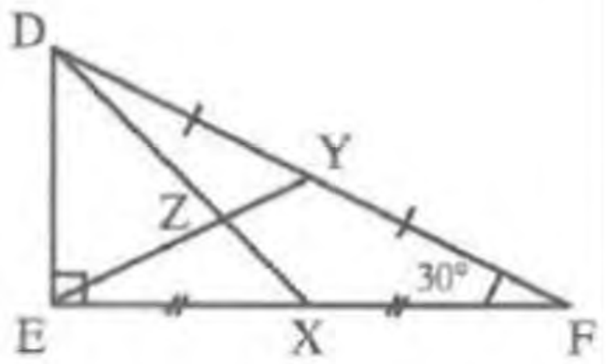
2 ***In the opposite figure :***

m (∠ DEF) = 90° , X and Y are the midpoints of $\overline{EF}$, $\overline{DF}$ respectively , m (∠ F) = 30°

DF = 12 , XZ = 2.5 find the perimeter of Δ DEZ



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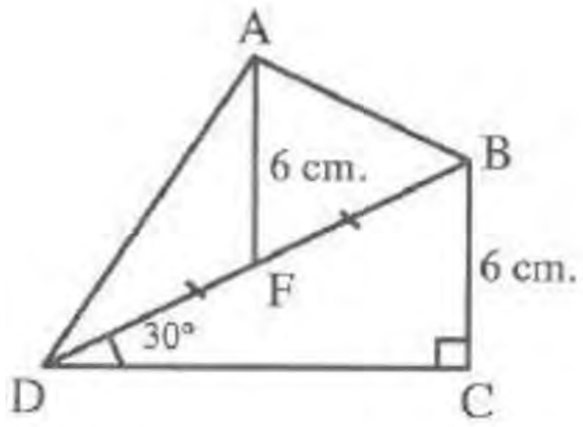
3 ***In the opposite figure :***

m (∠ C) = 90° , $\overline{AF}$ is a median of Δ ABD , m (∠ BDC) = 30°

BC = AF = 6 cm.

First : Find the length of $\overline{BD}$

Second : Prove that m (∠ BAD) = 90°



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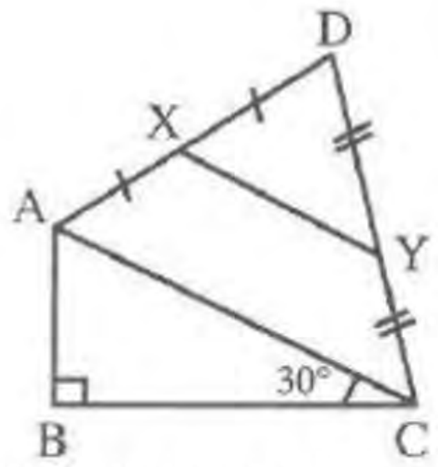
4 ***In the opposite figure :***

m (∠ ABC) = 90° , m (∠ ACB) = 30° , Y and X are the midpoints of $\overline{CD}$ and $\overline{AD}$ respectively.

Prove that : XY = AB



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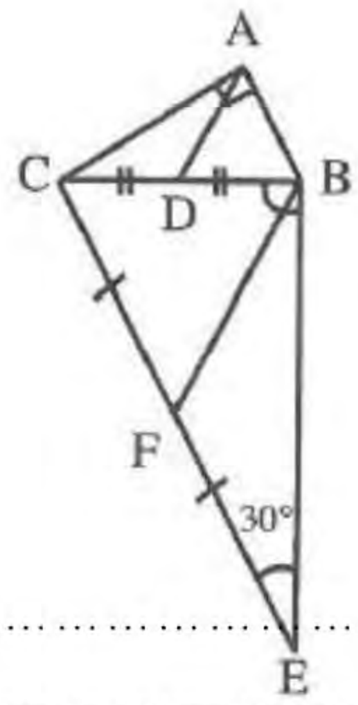
5 ***In the opposite figure :***



m (∠ BAC) = m (∠ CBE) = 90°

m (∠ BEC) = 30° , D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively

Prove that : $AD = \frac{1}{2}$ BF

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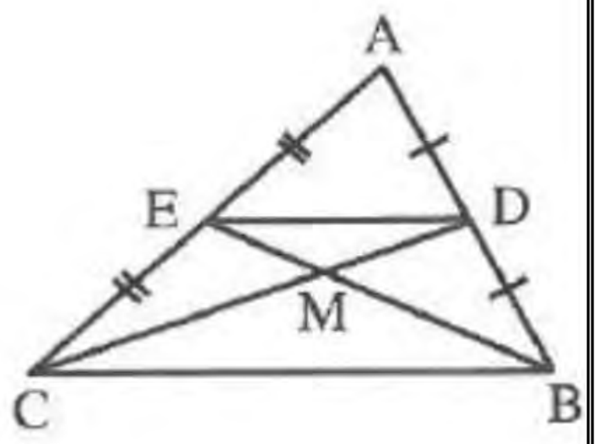
6 ***In the opposite figure :***



D and E are the midpoint of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 10 cm. , MB = 5 cm. , MC = 6 cm.

Find the perimeter of Δ MDE

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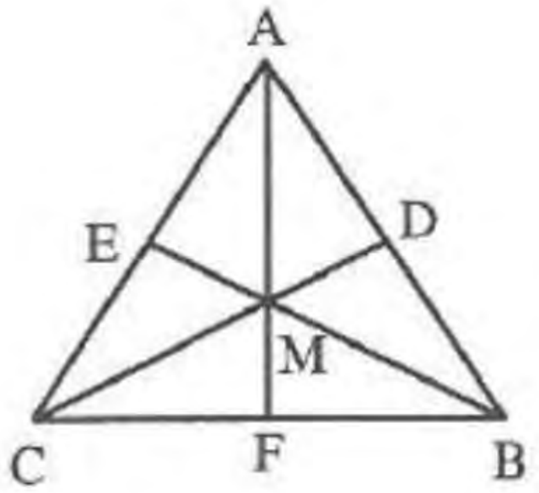
7 ***In the opposite figure :***



If M is the point of intersection of the medians of Δ ABC where BE = 6 cm. , CD = 9 cm. and BF = 3.5 cm.

Find the perimeter of Δ MBC

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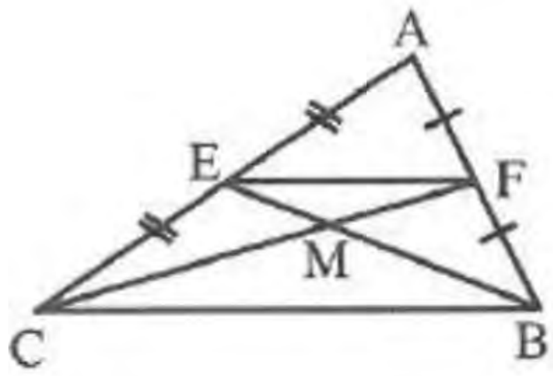
8 ***In the opposite figure :***

F and E are the midpoint of $\overline{AB}$ and $\overline{AC}$

in Δ ABC where BM = 5 cm. , CM = 6 cm.

Find the perimeter of Δ MEF



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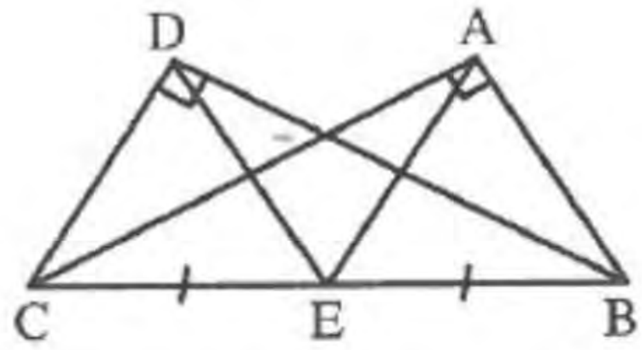
9 ***In the opposite figure :***

m ($\angle$ BAC) = m ($\angle$ BDC) = 90°

, E is the midpoint of $\overline{BC}$

Prove that : AE = DE



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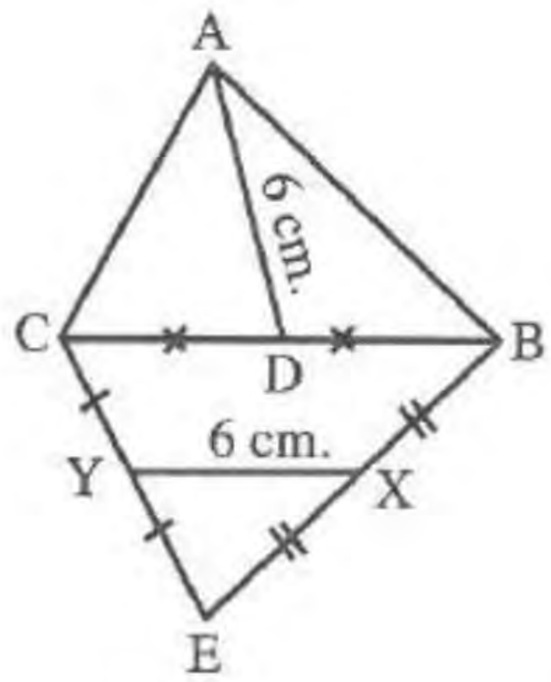
10 ***In the opposite figure :***

$\overline{AD}$ is a median of Δ ABC , X and Y

are the midpoints of $\overline{BE}$ and $\overline{CE}$ respectively

AD = XY = 6 cm.

Prove that : m ($\angle$ BAC) = 90°



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Answers

Algebra:

(1) Complete:

1) $Q \cup Q' = ..R....$

2) $-\sqrt{25} = \sqrt[3]{X}$,thenX=...-125...

3)if $x \in z, x < \sqrt{11} < x+1$,*then* $x = ...3...$

4) $\sqrt{7} \in [x, x+1], x \in z$ then x=...2..

5) the two consective integers which included the number $\sqrt{5}$ *are*....2..*and*....3....

6) the S.S of $8x^3 + 1 = -7$ in Ris....{-1]...

7)If $\sqrt[3]{x} = \sqrt{9}$, then x=....27....

8) $\sqrt[3]{x^6} = \sqrt{...x^4...}$

9) The set of non- negative real number=... $[0, \infty[$ ....(Write as intervals)

10) $R^+ = ...]0, \infty[....$ (Write as intervals)

11) the set of non-positive real numbers =... $]-\infty, 0]$ .....

12) s.s of $x^2 + 9 = 0$ in R *is*...φ....

13)*s.s of* $x^3 - 2 = 3$ in R *is*.....$\{\sqrt[3]{5}\}$...

14) $\left|\sqrt[3]{-27}\right| = \sqrt{....9.....}$

15)*s.s of* $x(x-2) = 0$ in R *is*..$\{0, 2\}$.....

16)*s.s of* $(x+3)(x-2) = 0$ in R *is*..$\{-3, 2\}$.....

17)if $x \in z, x < -\sqrt{11} < x+1$,*then* $x =$ - 4

18) the sum of the square roots of the number 25 =.....0....

2) **Choose the correct answer from those given :**

1 $\mathbb{R}$ = ...

(a) $\mathbb{Q} \cup \grave{\mathbb{Q}}$ (b) $\mathbb{Z}_+ \cup \mathbb{Z}_-$ (c) $\mathbb{R}_+ \cup \mathbb{R}_-$ (d) $\mathbb{N} \cup \mathbb{R}_-$

2 $\mathbb{Q} \cap \grave{\mathbb{Q}}$ = ...

(a) $\mathbb{Q}$ (b) $\grave{\mathbb{Q}}$ (c) $\mathbb{R}$ (d) $\varnothing$

3 $\mathbb{Q} \cup \grave{\mathbb{Q}}$ = ...

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\grave{\mathbb{Q}}$

4 $\mathbb{R}_+ \cap \mathbb{R}_-$ = ...

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}_-$

5 $\mathbb{R}_+ \cup \mathbb{R}_-$ = ...

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}_+$ (d) $\mathbb{R}^+$

6 $\mathbb{R} - \grave{\mathbb{Q}}$ = ...

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\{0\}$

7 $\mathbb{R} - \mathbb{Q}$ = ...

(a) $\grave{\mathbb{Q}}$ (b) $\mathbb{R}$ (c) $\varnothing$ (d) $\{0\}$

8 $\mathbb{R}_+ \cap \{-1, 0, 1\}$ = ...

(a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$

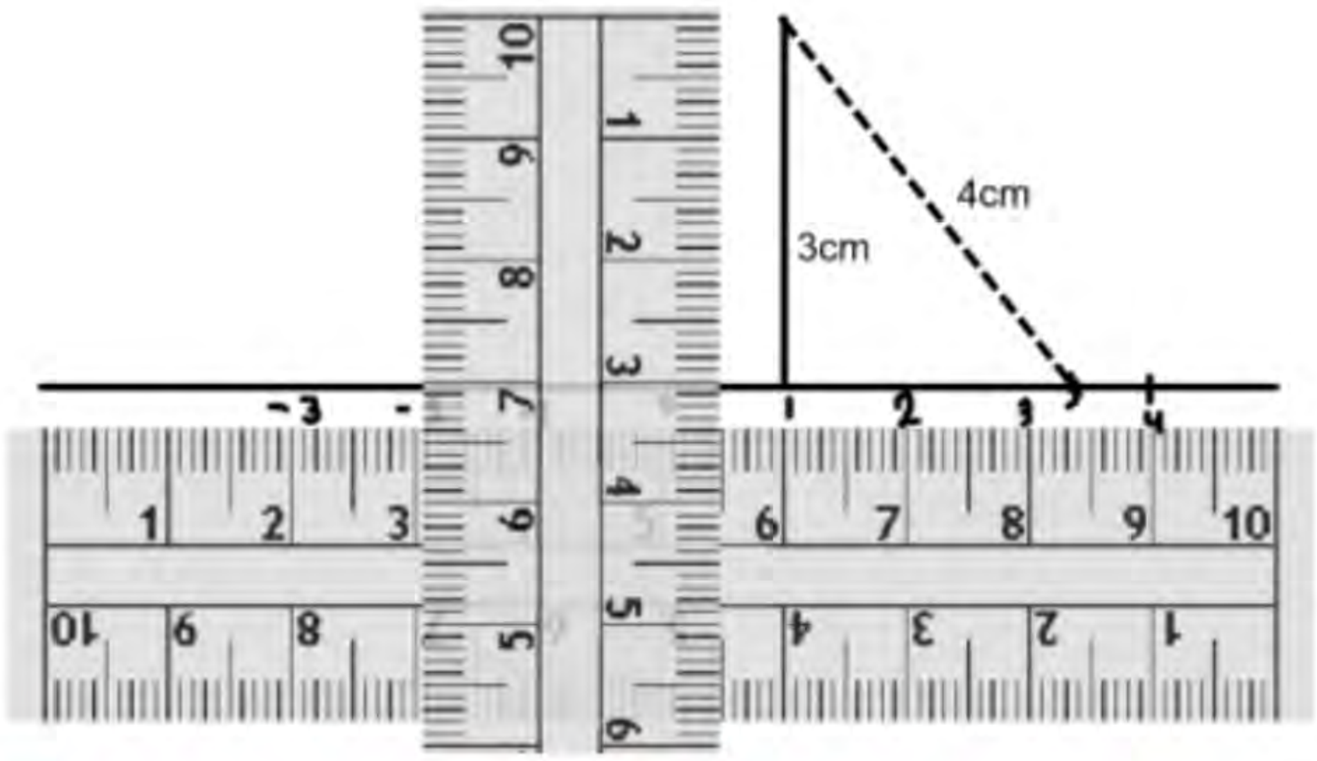
9 $\{x : x \in \mathbb{R}, x < 0\}$ = ...

(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

10 If x is a negative real number , then which of the following numbers is positive ?

(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

(5) Draw the number line and represent $1+\sqrt{7}$ on the number



(6)Find the S.S of each in Q the following:-

(1) $2X^2 - 3 = 47$

$\{5,-5\}$

(2) $X^3 - \frac{3}{8} = 3$

$\{\frac{3}{2}\}$

(3) $(X^3 - 1)(X^2 - 4) = 0$

$\{1, 2, -2\}$

(4) $(X+1)^3 = 125$

$\{4\}$

(7)Find the S.S of each in R the following:-

(1) $2X^3 + 4 = 16$

$\{\sqrt[3]{6}\}$

(2) $\frac{1}{8}X^3 - 3 = 5$

$\{4\}$

(3) $(X+1)^2 = 25$

$x+1 = \pm 5$

$x+1 = 5 \quad | \quad x+1 = -5$

$x = 4 \quad | \quad x = -6$

$s.s = \{4, -6\}$

(4) $(5X-2)^3 + 2 = 10$

$\{\frac{4}{5}\}$

Geometry:

First: Complete the following:

1) a) median b) one point c) 1 : 2

   d) concurrence e) $\frac{1}{2}$, 2 , $\frac{2}{3}$

2) a) 1 b) 4.5 c) 4

3) a) 6 cm b) 3 c) 3.6

4) a) $\frac{1}{2}$ the length of the hypotenuse

   b) the angle at this vertex is right

   c) equals half the length of hypotenuse

5) a) 4 b) 3 c) 3.5

6) a) 6 cm b) 4 cm c) 3 cm d) 9 cm

Second: Choose the correct answer from those given:

1) $\frac{3}{2}$ AM 2) 2 : 1 3) 4 cm 4) 3 cm

Third:

(1) Proof: ∵ In Δ ABC

m (∠ C) = 30°, m (∠ ABC) = 90°, D is the midpoint of $\overline{AC}$

∴ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC (1)

∴ AB = $\frac{1}{2}$ AC (2)

∴ AB = BD = AD

∴ Δ ABD is equilateral

(2) Proof: ∵ In Δ DEF

X is midpoint of $\overline{EF}$

∴ $\overline{DX}$ is a median, XZ = 2.5

∴ DZ = 2 ZX = 5 cm (1)

, Y is midpoint of $\overline{FD}$

∴ $\overline{EY}$ is median

EY = $\frac{1}{2}$ DF = 6 cm

EZ = $\frac{2}{3}$ EY = $\frac{2 \times 6}{3}$ = 4 cm (2)

∵ m (∠ F) = 30°

∴ DE = $\frac{1}{2}$ FD = 6 cm (3)

P. of Δ DEZ = 6 + 4 + 5 = 15 cm

(3) Proof: ∵ In Δ DCB

DC = 6 cm, m (∠ BDC) = 30°

∴ $BC = \frac{1}{2} DB$

∴ DB = 2 × 6 = 12 cm

∵ In Δ ABD

F is midpoint of $\overline{DB}$

, $AF = \frac{1}{2} BD$

∴ m ∠ BAD = 90°

(4) Proof: ∵ In Δ ACD

X, Y are midpoint of $\overline{AD}$, $\overline{DC}$

∴ $XY = \frac{1}{2} AC$ (1)

∵ In Δ ABC , m (∠ C) = 30° , m (∠ B) = 90°

∴ $AB = \frac{1}{2} AC$ (2)

From (1) , (2)

(6) Proof: ∵ In Δ ABC

E , D are midpoints of $\overline{AC}$, $\overline{AB}$

∴ $ED = \frac{1}{2} CB$

$ED = \frac{1}{2} \times 10 = 5$ cm (1)

∵ $\overline{CD}$ is median, $\overline{BE}$ is median, CM = 6 cm

∴ $MD = \frac{1}{2} CM = 3$ cm (2)

∵ MB = 5 cm

∴ $ME = \frac{1}{2} MB = \frac{1}{2} \times 5 = 2.5$ cm (3)

The perimeter of Δ MDE = 5 + 3 + 2.5 = 6.5 cm

(7) Proof: ∵ M is the point of intersection of the medians of Δ ABC

∴ F is midpoint of $\overline{BC}$, FB = 3.5 cm

∴ CB = 7 cm (1)

∵ BE = 6 cm

∴ $BM = \frac{2}{3} \times 6 = 4$ cm (2)

∵ CD = 9 cm

∴ $CM = \frac{2}{3} \times 9 = 6$ cm (3)

The perimeter of Δ MBC = 7 + 4 + 6 = 17 cm

5)

∵ In Δ ABC

∵ m (∠ A) = 90° , DC = DB

∴ $\overline{AD}$ is median

$AD = \frac{1}{2} CB$ (1)

∵ In Δ CBE

FC = FE

∴ $\overline{FB}$ is median

$FB = \frac{1}{2} CE$ (2)

, m (∠ E) = 30°

∴ $CB = \frac{1}{2} CE$ (3)

From (2) and (3)

∴ FB = CB

∴ $AD = \frac{1}{2} FB$

8) ∵ In Δ ABC

E , F are the midpoint of $\overline{AB}$ and $\overline{AC}$

∴ $EF = \frac{1}{2} BC = 5$ cm (1)

∵ BM = 5 cm

∴ $ME = \frac{1}{2} MB = 2.5$ cm (2)

∵ CM = 6 cm

∴ $MF = \frac{1}{2} MC = 3$ cm (3)

The perimeter of Δ MEF = 5 + 2.5 + 3 = 10.5 cm

9) **Proof:** ∵ In Δ ABC

m (∠ A) = 90° , CE = EB

∴ $\overline{AE}$ is median

$AE = \frac{1}{2} BC$ (1)

∵ In Δ CBD

m (∠ D) = 90°

$DE = \frac{1}{2} BC$ (2)

From (1) , (2)

AE = DE

10) In Δ CBE

X and Y are the midpoints of $\overline{BE}$ and $\overline{CE}$

∴ $XY = \frac{1}{2} CB$

CB = 2 × 6 = 12 cm

∵ In Δ ABC

, $\overline{AD}$ is median

∵ $AD = \frac{1}{2} CB$

∴ m (∠ BAC) = 90°

1-2 The cube root of a rational number

The cube root of the rational number "a" is the number whose cube equal to a

- The cube root of the rational number "a" is denoted by $\sqrt[3]{a}$
- The cube root of any number has the same sign of this number.
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$
- The cube root of a perfect cube rational number is also a rational number.

Find each of the following :

1) $\sqrt[3]{216}$

2) $\sqrt[3]{\frac{-8}{125}}$

3) $\sqrt[3]{0.064}$

4) $\sqrt[3]{1728}$

5) $-\sqrt[3]{0.216}$

6) $\sqrt[3]{-3\frac{3}{8}}$

Solve each of the following equations in ℚ :

7) $40x^3 - 1 = -136$

8) $(y-2)^3 = -343$

9) $27x^3 - 2 = 62$

10) $(5x-3)^3 - 2 = 6$

Notice that :

- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6
- The volume of the sphere = $\frac{4}{3}\pi r^3$
- 1 litre = 1000 cm^3

Find :

11) The length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.

12) The radius length of a sphere of volume $\frac{36}{125}\ \pi\ cm^3$

13) The diameter length of a sphere of volume equals 38808 cm^3 $(\pi \approx \frac{22}{7})$

14) **The length of the inner edge of a vessel in the shape of a cube with capacity 27 litres.**

15) **The length of the diameter of a sphere of volume 36 π cm^3**

1-3 The set of irrational numbers

- We studied that the number would be a rational number If it can be written in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$, $b \in \mathbb{Z}$ and $b \neq 0$
- **The square root of the perfect square rational number is a rational number.**
- **The cube root of the perfect cube rational number is a rational number.**

There is another set of numbers which are not rational numbers. This set is called "the set of irrational numbers" and it is denoted by $\mathbb{Q}'$

- $\mathbb{Q}$ and $\mathbb{Q}'$ are disjoint sets.
- $\mathbb{Q} \cap \mathbb{Q}' = \varnothing$

Show which of the following numbers belongs to $\mathbb{Q}$ and which of them belongs to $\mathbb{Q}'$:

1) $\sqrt{0.49}$ 2) $\sqrt{\frac{25}{49}}$ 3) $\sqrt{25} + \sqrt[3]{16}$ 4) $\sqrt[3]{-0.064}$

5) $\sqrt[3]{\frac{25}{49}}$ 6) 3 7) $\sqrt{9}$ 8) 5

9) $\sqrt{3}$ 10) -8 11) $\sqrt[3]{5}$ 12) $\sqrt[3]{-8}$

If $x \in \mathbb{Q}'$ find the S.S. of each of the following equations :

13) $x^2 = 5$ 14) $\frac{2}{5}x^2 = \frac{4}{25}$ 15) $(x^2 - 10)(x^3 - 4) = 0$

16) $x^3 = 7$

17) $64x^3 - 2 = -29$

18) $\frac{1}{2}x^2 - 5 = 3$

Remark

- Each irrational number lies between two rational numbers.
- Each irrational number can be represented by a point on the number line.

Prove that :

19) $\sqrt{3}$ lies between 1.7 and 1.8

20) $\sqrt[3]{12}$ lies between 2.2 and 2.3

21) $\sqrt{7}$ lies between 2.6 and 2.7

Find two consecutive integers such that

22) $\sqrt{13}$ **lies between them.**

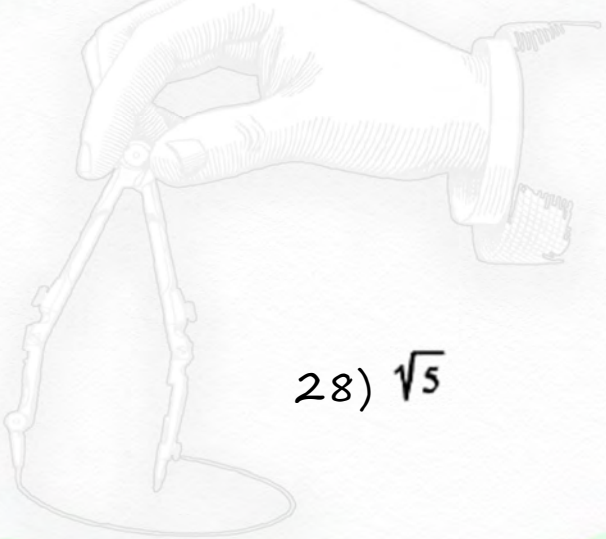
Determine the point which represents the number on the number line.

23) $\sqrt{7}$

24) $-\sqrt{7}$

25) $1+\sqrt{7}$

26) $2-\sqrt{7}$



27) $2\sqrt{7}$

28) $\sqrt{5}$

29) **Find the length of the diagonal of a square whose area = 5 cm^2.**

1-4 The set of real numbers

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

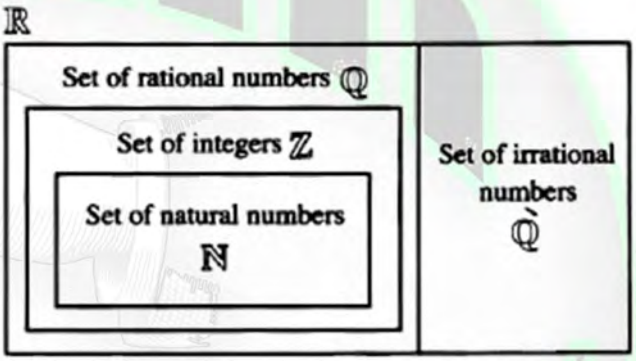
Noticing that :

- $\mathbb{Q} \cap \mathbb{Q} = \varnothing$

- **The opposite Venn diagram shows that :**

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

and $\grave{\mathbb{Q}} \subset \mathbb{R}$



Arrange the following numbers ascendingly :

1) $\sqrt{75}$, $\sqrt{68}$, $-\sqrt{45}$, -8 , 7 and $-\sqrt{32}$

.....

.....

.....

Complete each of the following using the suitable symbols > or < :

2) $\sqrt{2}$ 1

3) $-\sqrt[3]{7}$ ..... -2

4) $-\sqrt{3}$ ..... -1

5) $\sqrt{7}$ 2.6

6) $\sqrt[3]{9}$ 3

7) $-\sqrt[3]{16}$ ..... -2.52

Write three irrational numbers included between the two numbers

8) **11 and 12**

Find the S.S. in ℝ for each of the following equations :

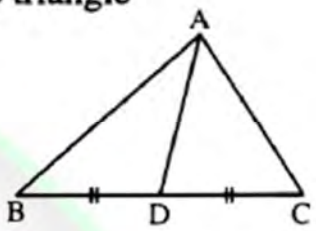
9) $3x^2 + 125 = 221$

10) $\frac{1}{6}x^3 - 8 = 28$

11) $2x^2 + 6 = 4$

4-2 Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.



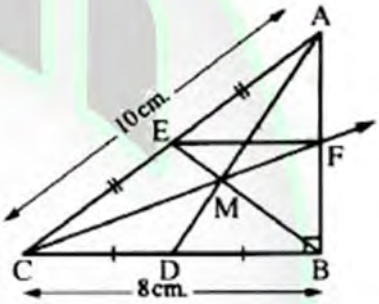
➢ **Theorem (1)**

The medians of a triangle are concurrent.

1) **In the opposite figure :**

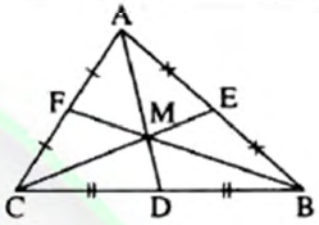
ABC is a right-angled triangle at B in which : AC = 10 cm. , BC = 8 cm. , D and E are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively where $\overline{AD} \cap \overline{BE} = \{M\}$

Draw $\overrightarrow{CM}$ to cut $\overline{AB}$ at F , Find the perimeter of Δ AFE



➢ **Theorem (2)**

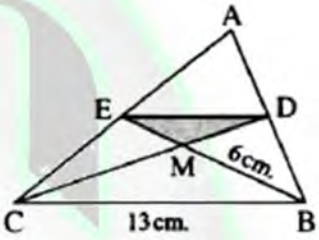
The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.



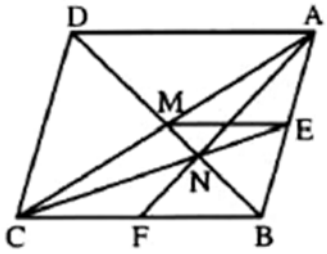
2) **In the opposite figure :**

In Δ ABC , $\overline{CD}$ and $\overline{BE}$ are two medians intersecting at M , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

Find the perimeter of Δ DME



3) **In the opposite figure :**



ABCD is a parallelogram ,

M is the point of intersection of its diagonals ,

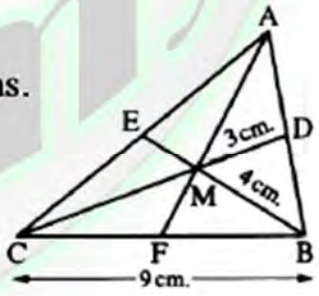
$N \in \overline{BM}$ where $BN = 2\,NM$, $\overline{AF} \cap \overline{BD} = \{N\}$ and $\overrightarrow{CN} \cap \overline{AB} = \{E\}$ **Prove that :** $EM = \frac{1}{2} BC$

4) **In the opposite figure :**

ABC is a triangle and M is the point of intersection of its medians.

If MD = 3 cm. , BM = 4 cm. and BC = 9 cm. ,

complete the following :

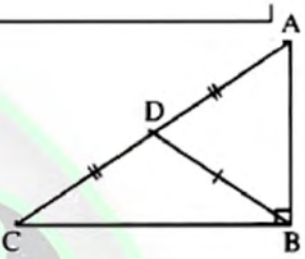


1 BF = ... cm. **2** MC = ... cm. **3** ME = ...cm.

4-3 (Follow) Median of triangle

➢ **Theorem (3)**

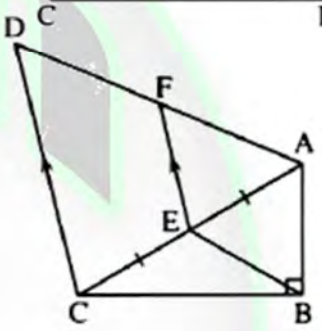
In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



1) **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ ABC) = 90° , AC = CD ,

E is the midpoint of $\overline{AC}$ and F ∈ $\overline{AD}$ such that $\overline{EF}$ // $\overline{CD}$



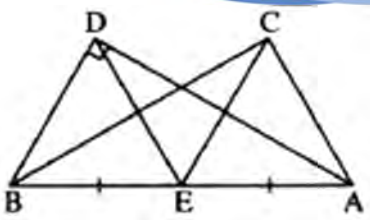
Prove that : BE = EF

➢ **The converse of theorem (3)**

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

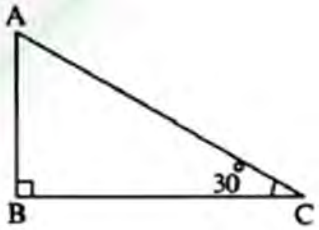
2) **In the opposite figure :**

ABD is a right-angled triangle at D , E is the midpoint of $\overline{AB}$ and CE = DE **Prove that :** m (∠ ACB) = 90°



➢ **Corollary**

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

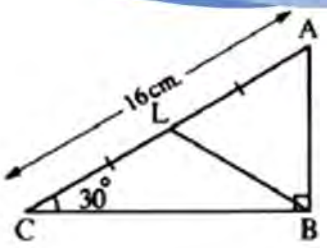


3) **In the opposite figure :**

ABC is a triangle in which m (∠ ABC) = 90° ,

m (∠ C) = 30° and AC = 16 cm. , L is the midpoint of $\overline{AC}$

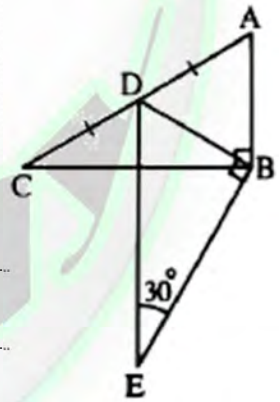
Find : the length of each of $\overline{AB}$ and $\overline{BL}$ and the perimeter of Δ ABL



4) **In the opposite figure :**

m (∠ ABC) = m (∠ DBE) = 90° , D is the midpoint of $\overline{AC}$

and m (∠ E) = 30° prove that : AC = DE



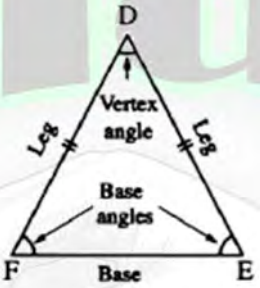
4-4 The isosceles triangle

Triangles are classified according to the lengths of their sides into three types which are :

➢ **Scalene triangle.**

$AB \neq BC \neq CA$

➢ **Isosceles triangle.**
(two sides are congruent).



$DE = DF$

➢ **Equilateral triangle.**
(three sides are congruent).

$XY = YZ = ZX$

➢ **Theorem (1)**

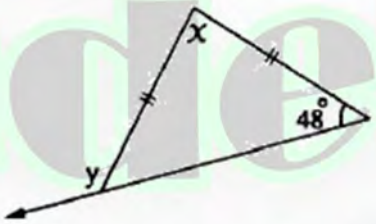
The base angles of the isosceles triangle are congruent.

In each of the following figures, find the values of the symbols used as a measure for the angle :

1)

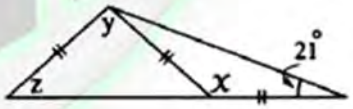
$x = \ldots\ldots^\circ$, $y = \ldots\ldots^\circ$

2)

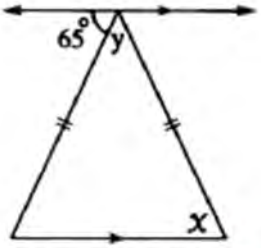


$x = \ldots\ldots^\circ$, $y = \ldots\ldots^\circ$

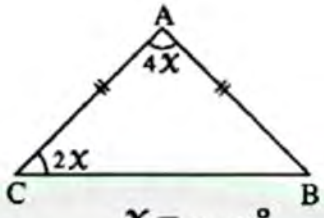
3)



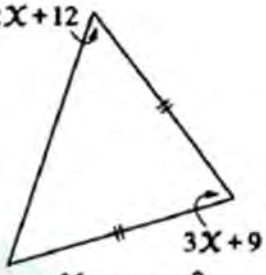
$x = \ldots\ldots^\circ$, $y = \ldots\ldots^\circ$, $z = \ldots\ldots^\circ$

4) 

x = ...° , y = ...°

5) 

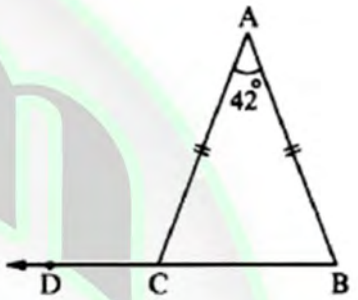
x = ...°

6) 

x = ...°

7) **In the opposite figure :**

$AB = AC$, $m(\angle A) = 42°$ and $D \in \overrightarrow{BC}$ **Find :** $m(\angle ACD)$



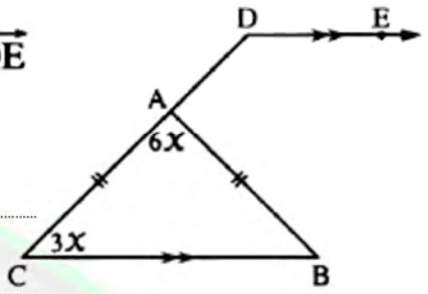
8) **In the opposite figure :**

$B \in \overline{DE}$, $C \in \overline{DE}$, $AB = AC$ and $BD = CE$ **Prove that :** $AD = AE$

9) **In the opposite figure :**

$AB = AC$, $m(\angle BAC) = 6x$, $m(\angle C) = 3x$ and $\overline{BC} // \overrightarrow{DE}$

Find : **1** The value of x **2** $m(\angle EDA)$

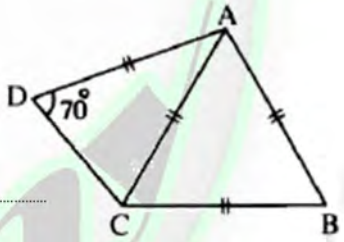


➢ **Corollary**

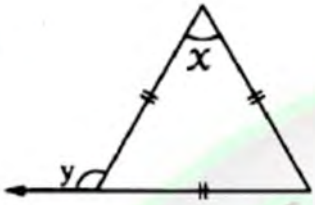
If the triangle is equilateral , then it is equiangular where each angle measure is 60°

10) **In the opposite figure :** $AB = BC = CA = AD$

$m(\angle D) = 70°$ **Find :** **1** $m(\angle BCD)$ **2** $m(\angle BAD)$



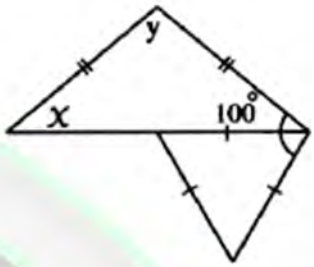
In each of the following figures , find the values of the symbols used as a measure for the angle :

11) 

x = ...° , y = ...°

12) 

x = ...° , y = ...°

13) 

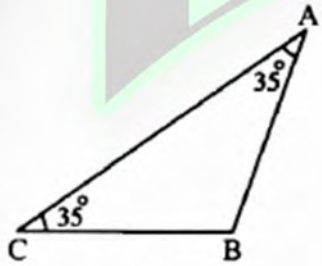
x = ...° , y = ...°

4-5 The converse of the isosceles triangle theorem

➢ **Theorem (2)**

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

In each of the following figures , write the equal sides in length:

1) 

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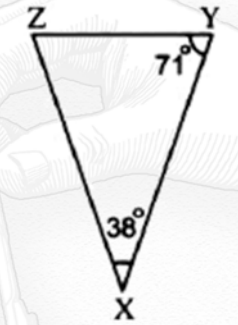
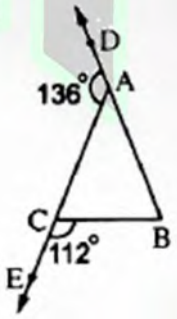
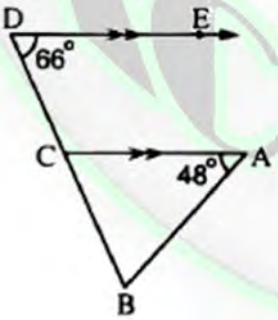
2) 

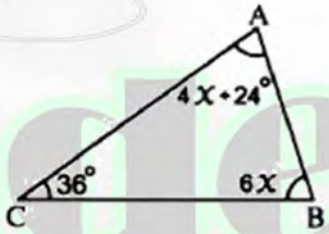
..... = ...

3) 

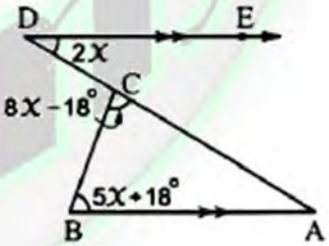
..... = ...

4) 

..... = ...

5) 

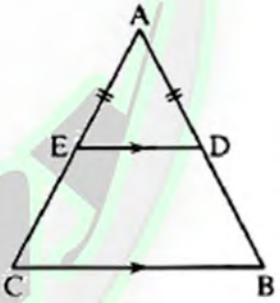
..... = ...

6) 

..... = ...

7) **ABC is a triangle in which m (∠ A) = 2 m (∠ B) = 72°**
Prove that : Δ ABC is an isosceles triangle.

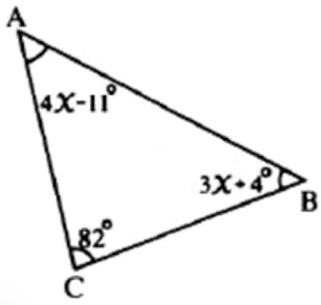
8) **In the opposite figure :** $D \in \overline{AB}$ and $E \in \overline{AC}$
where AD = AE and $\overline{DE} // \overline{BC}$ **Prove that :** DB = EC



9) **In the opposite figure :**

If $m(\angle A) = 4x - 11°$, $m(\angle B) = 3x + 4°$, $m(\angle C) = 82°$

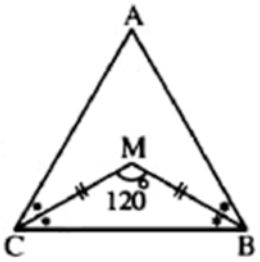
Prove that : ΔABC is an isosceles triangle.



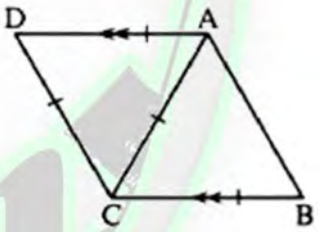
➢ **Corollary**

If the angles of a triangle are congruent, then the triangle is equilateral.

10) **In the opposite figure :** $\overrightarrow{BM}$ **bisects** $\angle B$, $\overrightarrow{CM}$ **bisects** $\angle C$,
$MB = MC$ **and** $m(\angle BMC) = 120°$
Prove that : ΔABC **is an equilateral triangle.**



11) **In the opposite figure :** $AD = DC = CB = CA$, $\overline{AD} // \overline{BC}$
prove that : ΔABC **is an equilateral triangle :**



1	$\sqrt[3]{x^6} = \sqrt{\ldots\ldots}$	(x^3 , x^2 , x , x^4)
2	The $S.S$ of the equation : $x(x^2-1)=0$ in R is	($\{0\}$, $\{1\}$, $\{-1\}$, $\{0,-1,1\}$)
3	$\mathbb{R} =$	($\mathbb{Q} \cap \mathbb{Q}'$, $\mathbb{R}_+ \cup \mathbb{R}_-$, $\mathbb{R}_+ \cap \mathbb{R}_-$, $\mathbb{Q} \cup \mathbb{Q}'$)
4	*The S.S of the equation* $x^2-9=0$ *in* $\mathbb{R}$ *is*	(3 , -3 , ± 3 , $\emptyset$)
5	$\mathbb{R}_+ \cup \mathbb{R}_- =$	($\emptyset$, $\{0\}$, $\mathbb{R}$, $\mathbb{R}-\{0\}$)
6	*The irrational number located between 2 and 3 is*	($\sqrt{10}$, $\sqrt{7}$, 2.5 , $\sqrt{3}$)
7	*The irrational number located between 3 and 4 is*	($\sqrt{6}$, $\sqrt{17}$, 3.5 , $\sqrt[3]{29}$)
8	$\sqrt{x^4} = \sqrt[3]{\ldots\ldots}$	(x^6 , x^4 , x^2 , x)
9	*The volume of sphere whose diameter length is* 6 *cm* = ... cm^3	(9π , 12π , 36π, 288π)
10	*if* $x < \sqrt{51} < x+1$, $x \in Z$, *then* $x =$	(8 , 7 , 6 , 5)
11	*if* π *is the ratio between the circumference of the circle and its diameter length , then* $\pi \in$	($\mathbb{Z}$, $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{Q}'$)
12	*The S.S in* $\mathbb{R}$ *for the equation :* $x^3+8=0$ *is*	($\{4\}$, $\{2\}$, $\emptyset$, $\{-2\}$)
13	$\{x : x \in \mathbb{R}, x > 0\} =$	($\mathbb{R}_-$, $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Q}$)
14	*The cube whose volume is* 216 cm^3 , *then the area of one of its face* = ... cm^2	(6 , 36 , 72 , 216)
15	$\sqrt[3]{9}$ ..... $\sqrt{4}$	($<$, $>$, $=$, $\leq$)

16	The $S.S$ of the equation $x^2 + 36 = 0$ in $\mathbb{R}$ is ..... ($\{6\}$, $\{-6\}$, $\{6, -6\}$, $\emptyset$)
17	*if* $\frac{x}{4} = \frac{16}{x^2}$, *then* x = ... (2 , 4 , 8 , 16)
18	*if the volume of a cube is* $64\ cm^3$, *then the length of its edge* = ... cm (8 , 4 , 16 , 64)
19	$(2 - \pi)$ ..... $\sqrt{(2-\pi)^2}$ (< , > , = , ≤)
20	*if the radius length of a sphere is* 6 cm , *then its volume is* ..... cm^3 (6π, 36π , 72π, 288π)
21	*if* $\sqrt[3]{x} = \sqrt{16}$, *then* x = ... (4 , -4 , 64 , -64)
22	$\mathbb{Q} \cap \mathbb{Q}'$ = ... ($\mathbb{Q}$, $\mathbb{R}$, $\emptyset$, $\mathbb{Q}'$)
23	$\{x : x \in \mathbb{R}, x \leq 0\}$ = ... ($\mathbb{R}_-$, $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{R}_- \cup \{0\}$)
24	*The irrational number in the following is* ..... ($\sqrt{\frac{1}{4}}$, $\sqrt{\frac{4}{9}}$, $\sqrt{2}$, $\sqrt[3]{8}$)

1	*The cube whose volume is* $8\ cm^3$, *then the sum of the lengths of its edges* = ...
2	*if* $\sqrt[3]{x} = -5$, *then* x = ...
3	*if* $x < \sqrt{51} < x + 1$, $x \in Z$, *then* x = ...
4	*if the volume of a sphere* = $\frac{9}{16}\pi$, *then its radius* = ... cm
5	$\sqrt[3]{\ldots\ldots} = -\sqrt{4}$
6	*if* $x < \sqrt{19} < x + 1$, $x \in \mathbb{Z}$, *then* x = ...
7	The $S.S$ of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is ..... , $x \in \mathbb{R}$

8	$\mathbb{R}_+ \cup \mathbb{R}_-$ = ...
9	*The S.S of the equation* $x^2 - 5 = 0$ *is* ..... *where* $x \in \mathbb{R}$
10	*The two consecutive integers which include the number* $\sqrt{5}$ *between them are* ..... *and* .....
11	*A square , its area* $50\ cm^2$ *, then length of its diagonal =* .....
12	*if the volume of a cube* $= 64\ cm^3$ *, then its lateral area =* ..... cm^2
13	$\sqrt[3]{27\ a^{12}}$ = ...
14	*if* $x \in \mathbb{Z}$ *and* $x < \sqrt[3]{29} < x + 1$ *, then* x = ...
15	*The S.S of* $x^3 + 9 = 0$ *in* $\mathbb{R}$ *is* .....
16	$R \cap R_-$ = ...
17	*A cube of edge length* $3\ cm$ *, then its volume* = ... cm^3
19	The *S.S of the equation :* $(x - \sqrt{5})(x + \sqrt{3}) = 0$ in $\mathbb{Q}'$ *is* .....
20	*if* $8 = \sqrt[3]{x}$ *, then* x = ...
21	$\mathbb{R} - \mathbb{Q}$ = ...
22	$\sqrt[3]{125} = \sqrt{\ldots\ldots}$
23	$\mathbb{R} - \mathbb{R}_-$ = ...
24	*The volume of a cube is* $27\ cm^3$ *, then the area of one of its faces is* ..... cm
25	$\mathbb{R}$ = ... $\cup$ ..... $\cup$ .....

1	**Find the S.S in $\mathbb{R}$: $2x^3 - 1 = 53$** ....
2	**Find the S.S in $\mathbb{R}$: $8x^3 + 7 = 8$** ....
3	**Find the S.S in $\mathbb{R}$: $(5x - 3)^3 = 8$** ....
4	**Find the S.S in $\mathbb{R}$: $(x - 3)^3 = 5$** ....
5	**Find the S.S in $\mathbb{R}$: $2x^2 - 6 = 4$** ....
6	**Prove that : $\sqrt{5}$ is included between 2.2 and 2.3** ....
7	**Prove that : $\sqrt[3]{15}$ is included between 2.4 and 2.5** ....
8	**Determine the point which represent the number $\sqrt{5}$ on the number line**

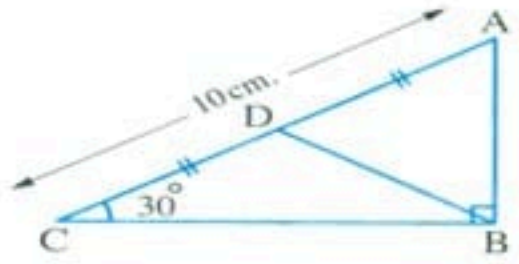
1	$in\ triangle\ ABC\ ,if\ m(\angle C)=60\ ,m(\angle B)=90\ ,then\ AC=$ ..... ($2\,BC$, $2\,AB$, $\frac{1}{2}\,AB$, $\frac{1}{2}\,AB$)
2	$in\ \Delta\ ABC$, if $AB \perp BC$ and $AB=BC$, $then\ m(\angle A)=$ .....° (30 , 45 , 60 , 90)
3	$if\ AD\ is\ a\ median\ of\ \Delta\ ABC$, and M is the point of concurrence of the median , then $AD=$ ..... AM ($\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{2}$)
4	$In\ a\ triangle\ ABC\ if\ AC=BC\ and\ m(\angle C)=80\ ,then\ m(\angle A)=$ ..... (80 , 50 , 100 , 40)
5	$The\ measure\ of\ any\ exterior\ angle\ of\ an\ equilateral\ triangle\ =$ ..... (45 , 60 , 90 , 120)
6	If M is the point of intersection of the medians of $\Delta\ ABC$ and D is the midpoint of BC , then $AD=$ ..... ($2\,AM$, $3\,MD$, $\frac{2}{3}\,MD$, AM)
7	The point of intersection of the medians of the triangle divides each median in the ratio of ..... from the vertex (2 : 1 , 2 : 3 , 1 : 2 , 1 : 3)
8	$if\ \Delta\ ABC\ is\ a\ right\ angled\ at\ A$ and $AB=AC$, then $m(\angle B)=$ .....° (30 , 45 , 60 , 90)
9	$ABC\ is\ an\ isosceles\ triangle\ ,m(\angle A)=100\ ,then\ m(\angle B)=$ ..... (40 , 50 , 80 , 100)
10	$if\ AD\ is\ a\ median\ of\ \Delta\ ABC$ and M is the point of concurrence of the medians , then $AM=$ ..... AD ($\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{2}$, 2)
11	In any isosceles triangle , the type of the base angles is ..... (acute , right , obtuse , reflex)

12	*if the measure of the vertex of angle of an isosceles triangle is* 50 , *then the measure of one of its base angle is* ..... (65 , 45 , 55 , 70)
13	*in* $\Delta\ ABC$: *if* $m(\angle B) = 90$, $AB = \frac{1}{2}\ AC$, *then* $m(\angle C)$ = ... (60 , 30 , 180 , 45)
14	The medians of the triangle intersect at ..... (4 points , 3 points , 2 points , a point)
15	*if* $\Delta\ ABC$ is an equilateral triangle, then $m(\angle B)$ = ...° (30 , 60 , 70 , 90)
16	*The number of medians of the right angled triangle* = ... (*one* , *two* , *three* , *four*)

1	The point of intersection of the medians of the triangle divides each of them in the ratio ..... : 5 from the vertex
2	If the length of the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length , then .....
3	*in* $\Delta\ ABC$, $AB = AC$, $m(\angle B) = x + 30^\circ$, $m(\angle C) = 2x + 5^\circ$, then x = ...
4	*in* $\Delta\ ABC$, if D is the midpoint of BC and $AD = \frac{1}{2}\ BC$, then $m(\angle A)$ = ...
5	*The base angles of the isosceles triangle are* .....
6	ABC is aright angled triangle at B , $m(\angle C) = 30^\circ$, $AB = 5\ cm$,then AC = ...
7	*In the right angled triangle the length of the median drawn from the vertex of the right angle* = ...

8	$in\ \Delta\ ABC$, if $m\ (\angle A\ ) = 30^{\circ}$, m($\angle$ B) = 90° , then $AC =$ ..... BC
9	$The\ medians\ of\ triangle\ are$ .....
10	$in\ \Delta\ ABC$ if the point $X\ is\ the\ midpoint\ of\ BC$, then $AX\ is\ called$ .....
11	The length of the side which is opposite to the angle of measure 30° in the right angled triangle equals ..... the length of the hypotenuse
12	ABC is a triangle in which $AB = AC$ and $m\ (\angle A\ ) = 60^{\circ}$, if its perimeter = 18 cm , then BC = ... $cm.$
13	If the measure of one of the base angles of an isosceles triangle equals 50° , then the measure of the vertex angle equals .....
14	If the angles of a triangle are congruent , then the triangle is .....
15	$in\ \Delta\ ABC$, if $AB = AC$, $m(\angle A\ ) = 70^{\circ}$, so $m(\angle C\ ) =$ .....
16	The point of concurrence of the medians of the triangle divides each median in the ratio of ..... from the base
17	$if\ \Delta\ ABC$ is a right angled triangle at B , $m(\angle A\ ) = 30^{\circ}$, AC = 10 cm. , then $CB =$ ..... $Cm.$
18	The length of the median of the right angled triangle drawn from the vertex of the right angle equals ..... The length of the hypotenuse
19	$in\ \Delta\ ABC$, if the point D is the midpoint of AB and the point E is the midpoint of AC , then $DE =$ ..... BC
20	$in\ \Delta\ DEF$, $if\ DE = DF$, then $m(\angle E\ ) = m(\angle$ ... ...)
21	$The\ base\ angles\ of\ an\ isosceles\ triangle\ are$ .....

1 m (∠ ABC) = 90° , m (∠ C) = 30°
, AD = DC and AC = 10 cm.
Find : The perimeter of Δ ABD



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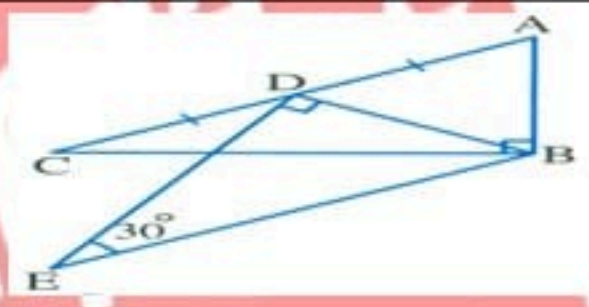
2 m (∠ ABC) = m (∠ BDE) = 90°
, m (∠ E) = 30°
, D is the midpoint of $\overline{AC}$
Prove that : AC = BE



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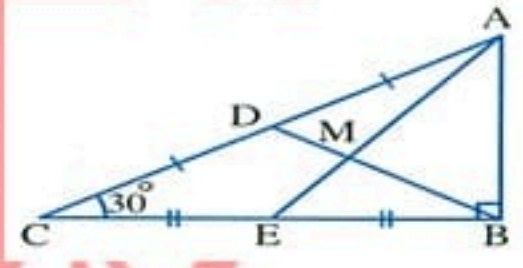
3 Δ ABC is right-angled at B
, m (∠ C) = 30° , D is the midpoint of $\overline{AC}$
, E is the midpoint of $\overline{BC}$, AC = 9 cm.
Find the length of each of : $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$



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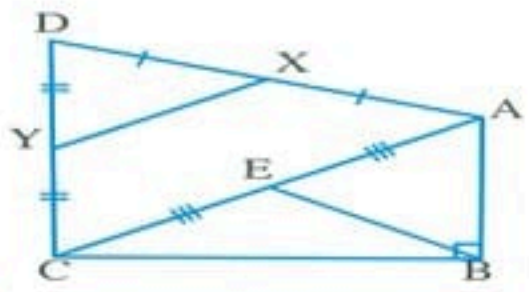
4 $m(\angle ABC) = 90°$

, E is the midpoint of $\overline{AC}$

and X , Y are the midpoints of $\overline{DA}$ and $\overline{DC}$

Prove that : $XY = BE$



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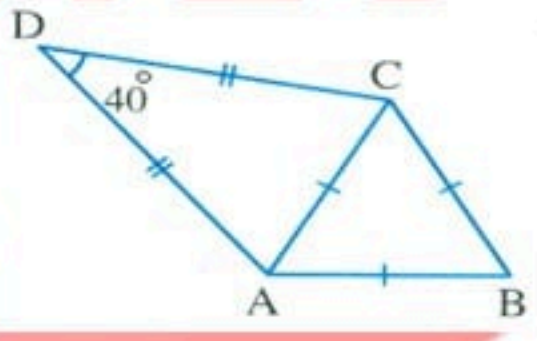
5 $m(\angle D) = 40°$

, $DA = DC$

and ΔABC is an equilateral triangle.

Find : $m(\angle DCB)$



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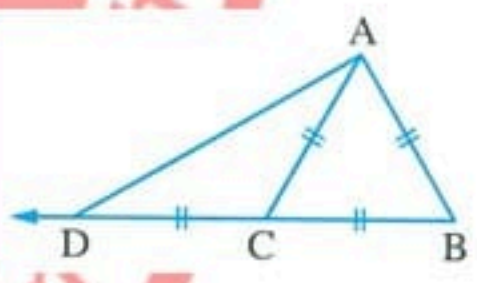
6 $AB = AC = CB = CD$

Prove that : $\overline{AB} \perp \overline{AD}$



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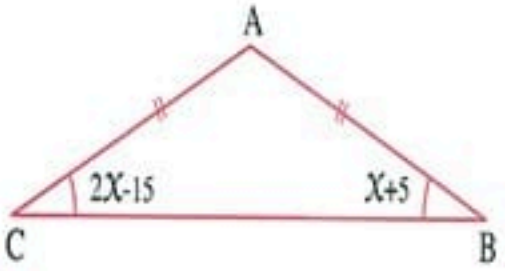
7 ABC is a triangle , AB = AC , m (∠ B) = $(\mathcal{X} + 5)°$

, m (∠ C) = $(2\mathcal{X} - 15)°$

Find : m (∠ A) (show all of your work)



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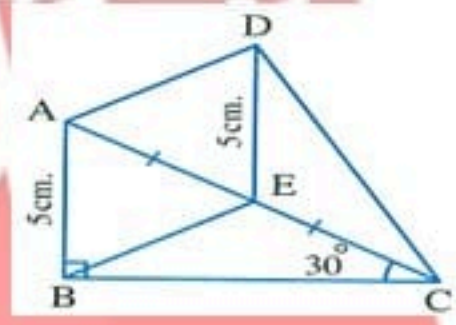
8 ABC is a right-angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, E is the midpoint of $\overline{AC}$, if DE = 5 cm.

, **prove that :** m (∠ ADC) = 90°



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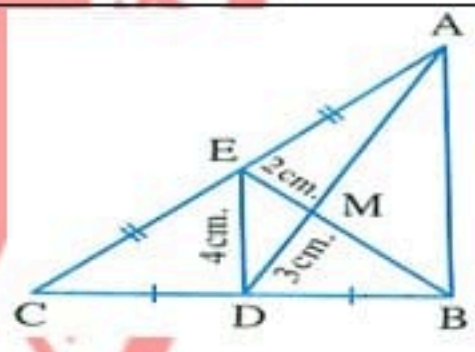
9 ***ABC is a triangle in which ME = 2 cm , MD = 3 cm***

DE = 4 cm , D , E are the midpoints of $\overline{BC}$ and $\overline{AC}$

respectively , find the perimeter of △ MAB



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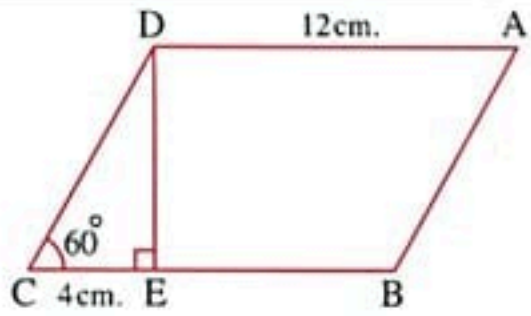
10 **In the opposite figure :**

ABCD is a parallelogram

, m (∠ C) = 60° , $\overline{DE} \perp \overline{BC}$

, AD = 12 cm. , CE = 4 cm.



Find with proof : The perimeter of the parallelogram ABCD

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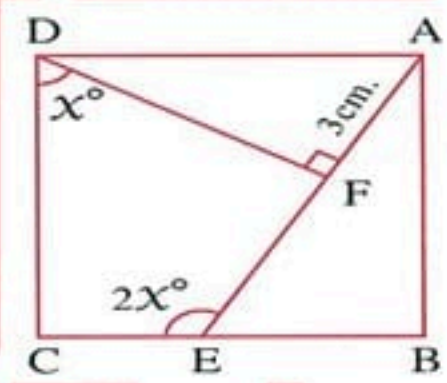
11 ABCD is a square , $E \in \overline{BC}$

where m (∠ FDC) = $\chi°$ and m (∠ FEC) = $2\chi°$

, $\overline{DF} \perp \overline{AE}$, AF = 3 cm.

Calculate : The area of the square ABCD



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